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# THE JOURNAL OF *Agricultural Economics Research*



United States  
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Economic  
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Service

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## *Articles*

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Nonlinear and Chaotic Dynamics: An Economist's Guide

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The Changing Structure of the U.S. Flour Milling Industry

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Determination of a Variable Price Support Schedule as Applied  
to Agricultural Production Control

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Nonfarm Prospects Under Agricultural Liberalization

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## *Book Reviews*

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A Taste of the Country: A Collection of Calvin Beale's Writings

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Three Faces of Power

---

Imperfect Competition and Political Economy: The New Trade Theory  
in Agricultural Trade Research

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U.S. Grain Policies and the World Market

---

The Political Economy of U.S. Agriculture: Challenges for the 1990s

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Risk Analysis: A Guide to Principles and Methods for Analyzing Health  
and Environmental Risks

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Agricultural Risk Management

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# In This Issue

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As a special feature of this issue, the Journal presents an article by Michael Weiss on the methodology of dynamical systems. Thirty years ago in *Econometrica*, Benoit Mandelbrot wrote the first in a series of articles challenging neoclassical economic theory's imitation of 19th-century energy physics. Those articles upset some of the fundamental notions of deterministic mechanics inherent in neoclassical theory. According to Mirkowski, Mandelbrot's contributions to economics were essentially dormant while he moved on to physics and meteorology. However, a significant and growing group of mathematically inclined economists is now taking an interest in, and extending, his ideas. Weiss's article provides the essentials of chaos for modeling dynamic behavior.

The three remaining articles more nearly reflect the traditional applied analyses of agricultural economics. Kim, Lin, and Leath examine the flour milling industry. Huang and Hyberg try their hand at fine-tuning agricultural price supports. Kilkenny models the off-farm effects of liberalizing agricultural trade policy. In the future, topics included in these papers, such as firm failures, commodity stocks, and employment fluctuations, may be extended with the insights of chaotic dynamics. In the meantime, our near-term analyses can simply assume that tracks and outcomes are not "sensitively dependent on initial conditions."

Kim, Lin, and Leath look at causes and consequences of the reduced number, expanded size, and changed location of flour mills in the United States. Size efficiencies and automation will increase the number of large mills, reduce the number of small mills and the total number of mills, and reduce employment. Mills have tended to move from sources of wheat supply toward markets for flour. Despite the estimated decline of mills from 203 in 1990 to 160 by the year 2000, the authors see no problem in meeting an increased demand for flour.

The Huang and Hyberg alternative to production controls for supply management is a form of price discrimination they call variable pricing support schedules. They propose to manage supplies of commodities with prices rather than with control of land use. The highest of the variable support prices would go to the first units of production, with the lowest prices for the last units at below expected market prices. Their programming model yielded price schedules that favor small farms over large farms, compared with the traditional mandatory production control programs.

Kilkenny looks beyond farms and agriculture as she examines the effects of liberalizing agricultural trade

policies. She argues that because freer trade reallocates resources throughout the economy, a computable general equilibrium model is useful in tracing the effects of a free trade policy. She employs a 10-sector CGE model to examine the effects of liberalizing policies under three adjustment scenarios. She allows government deficit reduction effects of liberalization to affect outcomes. Her analysis concludes that, if resulting savings reduce government deficits, additions to GNP, perhaps \$4.5 billion, will ensue from multilateral trade liberalization. Improvements in GNP from trade liberalization in the 1990's are expected to be less.

Several of the book reviews in this issue vary from our standard economics fare, thus reminding us that, notwithstanding the title of this journal, we do embrace all the social sciences. That eclecticism is well represented by Calvin Beale's diverse writings, collected under the title *A Taste of the Country*, insightfully and supportingly reviewed by Sonya Salamon.

Members of the economics club can reach beyond Walrasian optima, witness Kenneth Boulding's *Three Faces of Power*, reviewed by Dwight Gadsby. Boulding has presented the three dimensions of human relations—threat, exchange, and love—in many articles and books. Here, like Lasswell, he shows the relationship between political and economic force.

The other books reviewed in this issue are closer to conventional agricultural economics and policy. Daniel Pick recommends reading the collection of papers on imperfect competition and trade theory edited by Carter, McCalla, and Sharples. Robert Green is only moderately supportive of the collection of papers on grain policies and the world market, prepared by Roberts, Love, Field, and Klijn. Dave Ervin recommends the collection of papers on the political economy of agriculture, edited by Kramer, but emphasizes that the historical content is stronger than its empirical content.

Michael Wetzstein reviews two recent books on risk. The guide to principles and methods of risk analysis by Cochrane and Covello is directed primarily at health and environmental risks. Never mind, the methods and principles apply to the broad range of problems in which risk is an element of concern. Wetzstein says the guide is "an excellent foundation for any student interested in learning how risk analysis is currently undertaken." The book by Fleisher on agricultural risk management is a broad overview of techniques. As such, this short volume may lack some of the detail of other references, but it is an excellent text with which to enter the field of risk management.

Gene Wunderlich

# Nonlinear and Chaotic Dynamics: An Economist's Guide

Michael D. Weiss

**Abstract.** *In recent years, research in both mathematics and the applied sciences has produced a revolution in the understanding of nonlinear dynamical systems. Used widely in economics and other disciplines to model change over time, these systems are now known to be vulnerable to a kind of "chaotic," unpredictable behavior. This article places this revolution in historical context, discusses some of its implications for economic modeling, and explains many of the important mathematical ideas on which it is based.*

**Keywords.** *Limits to predictability, nonlinear and chaotic dynamical systems, structural stability of economic models, fractals.*

In the past two decades, the world of science has come to a fundamentally new understanding of the dynamics of phenomena that vary over time. Grounded in mathematical discovery, yet given empirical substance by evidence from a variety of disciplines, this new perspective has led to nothing less than a re-examination of the concept of the predictability of dynamic behavior. Our implicit confidence in the orderliness of dynamical systems, specifically of *nonlinear* dynamical systems, has not, it turns out, been entirely justified. Such systems are capable of behaving in ways that are far more erratic and unpredictable than once believed. Fittingly, the new ideas are said to concern *chaotic dynamics*, or, simply, *chaos*.

Economics is not immune from the implications of this new understanding. After all, our subject is replete with dynamic phenomena ranging from cattle cycles to stock market catastrophes to the back-and-forth interplay of advertising and product sales. Ideas related to the notion of chaotic behavior are now part of the basic mathematical toolkit needed for insightful dynamical modeling. Agricultural economists need to gain an understanding of these ideas just as they would any other significant mathematical contribution to their field. This article is intended to assist in this educational process.

What exactly has chaos theory revealed? To address this question, let us consider an economy, subject to change over time, whose state at time  $t$  can be described by a vector,  $v_t$ , of (say) 14 numbers (money supply at time  $t$ , inflation rate at time  $t$ , and so on). Formally, this vector is a point in the 14-dimensional

state space  $\mathbf{R}^{14}$  (where  $\mathbf{R}$  is the real number line). Suppose that the economy evolves deterministically in such a way that its state at any time uniquely determines its state at all later times. Then, if the initial position of the economy in  $\mathbf{R}^{14}$  at time 0 is  $v_0$ , the evolution of the economy through time will be represented by a path in  $\mathbf{R}^{14}$  starting at  $v_0$  and traced out by  $v_t$  as time,  $t$ , moves forward. This path, called the *orbit* generated by the initial position  $v_0$ , represents a "future history" of the system. Questions about the behavior of the economy over time are really questions about its orbits. We are often interested not so much in the near-term behavior of orbits as in their eventual behavior, as when we engage in long-range forecasting or study an economy's response to a new government policy or an unexpected shock after the initial period of adjustment has passed and the economy has settled down.

## Fractals, Sensitive Dependence, and Chaos

Scientists long have known that it is possible for a system's state space to contain an isolated, unstable point  $p$  such that different initial points near  $p$  can generate orbits with widely varying longrun behavior. (For example, a marble balanced on the tip of a cone is unstable in this sense.) What was unexpected, however, was the discovery that this type of instability can occur *throughout* the state space, sometimes actually at every point, but often in strangely patterned, fragmented subsets of the state space—subsets typically of noninteger dimension, called *fractals*. Once investigators knew what to look for, they found this phenomenon, termed *sensitive dependence on initial conditions*, to be widespread among nonlinear dynamical systems, even among the simplest ones. Though technical definitions vary, systems exhibiting this unstable behavior have generally come to be called "chaotic."

For chaotic systems, any error in specifying an initial point, even the most minute error (due to, say, computer rounding in the thousandth decimal place), can give rise to an orbit whose longrun behavior bears no resemblance to that of the orbit of the intended initial point. Since, in the real world, we can never specify a point with mathematically perfect precision, it follows that practical longrun prediction of the state of a chaotic system is impossible.

## Attractors

For a dynamical system, perhaps the most basic question is "where does the system go, and what does it do when it gets there?" In the earlier view of dynamical

Weiss is an economist in the Commodity Economics Division, ERS. The author thanks John McClelland and other participants in the ERS Chaos Theory Seminar for many stimulating discussions on chaotic dynamics. Carlos Arnade, Richard Heifner, and an anonymous referee furnished helpful review comments.



systems, the place where the system went, the point set in the state space to which orbits converged (called an *attractor*), was usually assumed to be a geometrically uncomplicated object such as a closed curve or a single point. Economic modelers, for example, have often implicitly assumed that a dynamic economic process will ultimately achieve either an equilibrium, a cyclic pattern, or some other orderly behavior. However, another discovery of chaos theory has been that the attractor of a nonlinear system can be a bizarre, fractal set within which the system's state can flit endlessly in a chaotic, seemingly random manner.

Just as an economy can have two or more equilibria, a dynamical system can have two or more attractors. In such a case, the set of all initial points whose orbits converge to a particular attractor is called a *basin of attraction*. A recent finding has been that the *boundary* between competing basins of attraction can be a fractal even when the attractors themselves are unexceptional sets. A type of sensitivity to the initial condition can operate here too: the slightest movement away from an initial point lying in one basin of attraction may move the system to a new basin of attraction and thus cause it to evolve toward a new attractor.

Chaotic behavior within a working model would be easier to recognize if all orbits initiating near an erratic orbit were also erratic. However, the potentially fractal structure of the region of sensitive dependence can allow initial points whose orbits behave "sensibly" and initial points whose orbits are erratic to coexist inseparably in the state space like two intermingled clouds of dust. Thus, simulation of a model at a few trial points cannot rule out the possibility of chaotic dynamics. Rather, we need a deeper understanding of the mathematical properties of our models. Nor can chaotic dynamics be dismissed as arising only in a few quirky special cases. As we shall see, it arises even when the system's law of motion is a simple quadratic.

## The Discovery of Chaos

Recent years have witnessed an explosion of interest and activity in the area of chaotic dynamics. What accounts for this new visibility, which extends even beyond the research community into the public media? To provide an answer, we briefly trace the historical development of the subject.

The first recognition of chaotic dynamics is attributed to Henri Poincaré, a French mathematician whose work on celestial mechanics around the turn of the century helped found the study of dynamical systems, systems in which some structure (perhaps a solar system, perhaps—as now understood—an economy) changes over time according to predetermined rules. Poincaré foresaw the potential for unpredictability in dynamical systems whose equations of motion were nonlinear. However, neither the mathematical theory

nor the imaging techniques available at the time permitted him to explore his intuitions fully.

Following Poincaré's work and that of the American mathematician G.D. Birkhoff in the early part of this century, and despite continuing interest in the Soviet Union, the subject of dynamical systems fell into relative obscurity. During this period, there was some awareness among mathematicians, scientists, and engineers that nonlinear systems were capable of erratic behavior. However, examples of such behavior were ignored, classified as "noise," or dismissed as aberrations. The idea that these phenomena were characteristic of nonlinear dynamical systems and that it was the well-behaved, textbook examples that were the special cases had not yet taken root.

Then, in the 1960's and 1970's, there was a flurry of activity in dynamical systems by both mathematicians and scientists working entirely independently. Mathematician Stephen Smale turned his attention to the subject and used the techniques of modern differential topology to create rigorous theoretical models of chaotic dynamics. Meteorologist Edward Lorenz discovered that a simple system of equations he had devised to simulate the earth's weather on a primitive computer displayed a surprising type of sensitivity: the slightest change in the initial conditions eventually would lead to weather patterns bearing no resemblance to those generated in the original run.

Biologist Robert May used the logistic difference equation  $x_{n+1} = rx_n(1-x_n)$  to model population level,  $x$ , over successive time periods. He observed that for some choices of the growth rate parameter,  $r$ , the population level would converge, for other choices it would cycle among a few values, and for still others it would fluctuate seemingly randomly, never achieving either a steady state or any discernible repeating pattern. When he attempted to graph the population level against the growth rate parameter, he observed a strangely patterned, fragmented set of points.

Physicist Mitchell Feigenbaum investigated the behavior of dynamical systems whose equations of motion arise from unimodal (hill-shaped) functions. He noticed that certain parameter values that sent the system into repeating cycles always displayed the same numerically precise pattern: no matter which dynamical system was examined, the ratios of successive distances between these parameter values always converged to the same constant, 4.66920... Feigenbaum had discovered a universal property of a class of nonlinear dynamical systems. His discovery ultimately clarified how systems can evolve toward chaos.

Thus, as these and other examples demonstrate, while mathematicians were developing the theory of nonlinear and chaotic dynamics, scientists in diverse disci-

plines were witnessing and discovering chaotic phenomena for themselves. Ultimately, researchers learned of one another's findings and recognized their common origin.

The role of the computer in the emergence of the contemporary understanding of dynamical systems is difficult to exaggerate. As we now realize, even the simplest systems can generate bewilderingly complicated behavior. The development of modern computer power and graphics seems to have been necessary before researchers could put the full picture of nonlinear and chaotic dynamics, quite literally, into focus.

## The Mathematics of Chaos

We now explain some of the basic mathematical ideas involved in nonlinear dynamics and chaos. We also adopt a slightly different perspective. In the above discussion, we have implicitly portrayed dynamical systems as being in motion in continuous time. However, the equations of motion of such systems typically involve differential equations, and a proper treatment often requires advanced mathematical machinery. It is generally much easier to work with (and to understand) discrete-time systems, in which time takes only integer values representing successive time periods. Let us shift our attention to these systems.

When the law of motion of a discrete dynamical system is unchanging over time, the movement of the system through time can be understood as a process of iterating a function. To establish this point, consider a typical dynamic economic computer model,  $M$ , having  $k$  endogenous variables. To start the model running, one enters an initial condition vector,  $v_0$ , of  $k$  numbers. The model computes an output vector,  $M(v_0)$ , containing the new values of the  $k$  endogenous variables at the end of the first time period. The model then acts on  $M(v_0)$  and computes a new output vector,  $M(M(v_0))$ , describing the economy at the end of the second time period. Successive output vectors are computed in the same manner. Note that the model itself, the law of motion, remains unchanged during this process. In effect,  $M$  acts as a function, mapping  $k$ -vectors to new  $k$ -vectors, applying itself iteratively to the last-computed function value. The state space of the economy is the  $k$ -dimensional space  $\mathbf{R}^k$ , and, for each initial condition vector  $v_0$ , there is a corresponding orbit,  $v_0, M(v_0), M(M(v_0)), M(M(M(v_0))), \dots$ , describing the future evolution of the economy.

More generally, consider any function  $f$ . If  $f$  maps its domain (the set of all  $x$  for which  $f(x)$  is defined) into itself, then, for each  $x_0$  in the domain of  $f$ , the sequence  $x_0, f(x_0), f(f(x_0)), f(f(f(x_0))), \dots$  is well-defined and may be considered an orbit of a dynamical system determined by  $f$  through iteration.

Henceforth, for brevity, we denote by:

$$f^n,$$

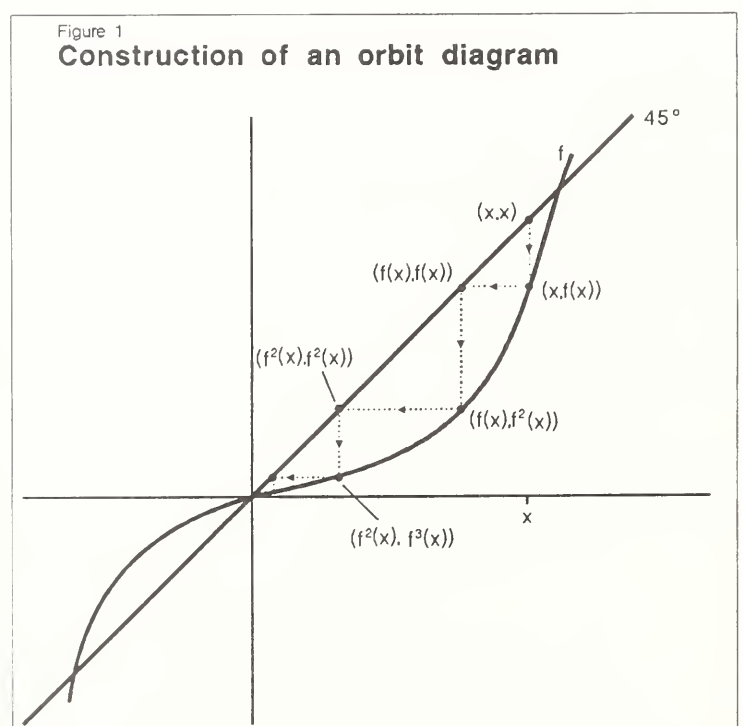
the  $n$ th iterate of a function  $f$ . Thus,  $f^1(x) = f(x)$ ,  $f^2(x) = f(f(x))$ ,  $f^3(x) = f(f(f(x)))$ , and so on. By convention,  $f^0(x) = x$ . Of course,  $f^n$  is itself a function. It should not be confused with the  $n$ th *derivative* of  $f$ , which is customarily denoted:

$$f^{(n)}.$$

## Orbit Diagrams

Fortunately for expository purposes, many of the important features of dynamical systems are present in one-dimensional systems. In fact, one of the important findings of chaos research has been that discrete dynamical systems generated by iteration of even the most elementary nonlinear scalar functions are capable of chaotic behavior. Thus, we shall concentrate on functions that operate on the number line.

For such functions, there is a particularly convenient technique for diagramming orbits. Consider a function  $f$  and an initial point  $x$  (fig. 1). Beginning at the point  $(x, x)$  on the  $45^\circ$  line, draw a dotted line vertically to the graph of  $f$ ; the point of intersection will be  $(x, f(x))$ . From that point, draw a dotted line horizontally to the  $45^\circ$  line; the point of intersection will be  $(f(x), f(x))$ . From there, draw a dotted line vertically to the graph of  $f$ ; the point of intersection will be  $(f(x), f^2(x))$ . Continue this pattern of moving vertically to the graph of  $f$  and then horizontally to the  $45^\circ$  line. The resulting display, called an *orbit diagram*, shows the behavior of the orbit originating at  $x$ . In particular, the orbit may





be visualized from the intersection points marked on the 45° line; the dotted lines indicate the direction of motion of the system. Of course, the points  $(x, x)$ ,  $(f(x), f(x))$ ,  $(f^2(x), f^2(x))$ , ... only *look like* the orbit. They reside in the plane, whereas the actual orbit, consisting of the numbers  $x$ ,  $f(x)$ ,  $f^2(x)$ , ..., resides in the state space, that is, in the number line.

## Dynamics of Linear Systems

Though the basic focus of this paper is nonlinear dynamics, examination of linear systems provides essential intuition about nonlinear ones. Thus, we begin with an exhaustive treatment of the linear case.

Choose any numbers  $a, b$ , and consider the function  $g$  defined by  $g(x) = ax + b$ . To compute a typical orbit of  $g$ , observe that:

$$\begin{aligned} g^2(x) &= a(ax+b) + b \\ &= a^2x + b(1+a), \\ g^3(x) &= a[a^2x + b(1+a)] + b \\ &= a^3x + b(1+a+a^2), \\ g^4(x) &= a^4x + b(1+a+a^2+a^3), \end{aligned}$$

and, in general,  $g^n(x) = a^n x + b(1+a+a^2+a^3+\dots+a^{n-1})$ . If  $a = 1$ , then

$$g^n(x) = x + bn.$$

However, if  $a \neq 1$ , the formula for the sum of a geometric series gives:

$$\begin{aligned} g^n(x) &= a^n x + b \left[ \frac{1-a^n}{1-a} \right] \\ &= a^n \left[ x - \frac{b}{1-a} \right] + \frac{b}{1-a}. \end{aligned}$$

Note that when  $a$  is nonnegative,  $a^n$  remains nonnegative, while when  $a$  is negative,  $a^n$  alternates between negative and positive. In particular, when  $a = -1$ ,  $a^n$  alternates between  $-1$  and  $1$ . When  $a \neq \pm 1$ , the distance between  $a^n$  and  $0$  either converges monotonically to  $0$  or diverges monotonically to  $\infty$  as  $n \rightarrow \infty$  according to whether  $|a| < 1$  or  $|a| > 1$ . Using these facts, we now analyze the behavior of all the orbits generated by  $g$ , according to the various possibilities for the structural parameters  $a$  and  $b$  and the initial point  $x$ . We shall find it convenient to organize our analysis around the possible value of  $a$ . We distinguish seven cases: (1)  $a < -1$ ; (2)  $a = -1$ ; (3)  $-1 < a < 0$ ; (4)  $a = 0$ ; (5)  $0 < a < 1$ ; (6)  $a = 1$ ; and (7)  $a > 1$ . Within each of these cases, we consider all possible values of the remaining structural parameter  $b$  and the initial point  $x$ , and we determine the longrun behavior of the orbit originating at  $x$  when  $g$  has structural parameters  $a$  and  $b$ .

Let us first dispense with the case  $a = 1$ . In this case, if  $b = 0$ , then every  $x$  is a fixed point of  $g$  (that is,  $g(x) = x$ ), and (since then, also  $g^n(x) = x$ ) the system always remains at any initial point. In contrast, if  $b \neq 0$ , then no  $x$  is a fixed point of  $g$ ; indeed, for any initial point  $x$ ,  $g^n(x)$  diverges monotonically as  $n \rightarrow \infty$  to either  $\infty$  or  $-\infty$  according to whether  $b > 0$  or  $b < 0$ .

In discussing the six remaining cases, that is, the cases in which  $a \neq 1$ , I take  $b$  to be an arbitrary number. In these cases,  $g$  has exactly one fixed point,  $b/(1-a)$ , and any orbit originating there remains there. I next examine the behavior of orbits originating at points other than  $b/(1-a)$ . For this purpose, I assume that the initial point  $x$  is an arbitrary number distinct from  $b/(1-a)$ .

If  $a < -1$ , then  $g^n(x)$  has no finite or infinite limit. Rather, it eventually alternates between positive and negative numbers as its absolute value diverges monotonically to  $\infty$ .

If  $a = -1$ , the fixed point  $b/(1-a)$  equals  $b/2$ , and:

$$\begin{aligned} g^n(x) &= (-1)^n(x-b/2) + b/2 \\ &= \begin{cases} b-x & \text{if } n \text{ is odd} \\ x & \text{if } n \text{ is even.} \end{cases} \end{aligned}$$

Thus,  $g^n(x)$  alternates endlessly between the (distinct) numbers  $b-x$  and  $x$ .

If  $-1 < a < 0$ ,  $g^n(x)$  converges to  $b/(1-a)$  while alternating above and below it.

If  $a = 0$ , then, for all  $n$ ,  $g^n(x) = b$ . Thus, the system moves from the initial point directly to  $b$  and remains there.

If  $0 < a < 1$ ,  $g^n(x)$  converges monotonically to  $b/(1-a)$ . The convergence is from above if  $x > b/(1-a)$  and from below if  $x < b/(1-a)$ .

Finally, if  $a > 1$ , then  $g^n(x)$  diverges monotonically, to  $\infty$  if  $x > b/(1-a)$  and to  $-\infty$  if  $x < b/(1-a)$ .

The possible behaviors of orbits in the one-dimensional linear system are illustrated in figures 2(a)–2(h). From these figures and the preceding discussion, two lessons emerge. First, the fixed point is often at the “center of the action”: it is to or from this point that orbits typically converge or diverge. Second, the slope parameter,  $a$ , plays a pivotal role in determining orbit dynamics. These two principles hold as well for nonlinear systems.

## Fixed Points and Periodic Points

It is not a coincidence that, in the linear system, convergent orbits always converge to a fixed point of the underlying function. In fact, this property holds in

Figure 2(a)

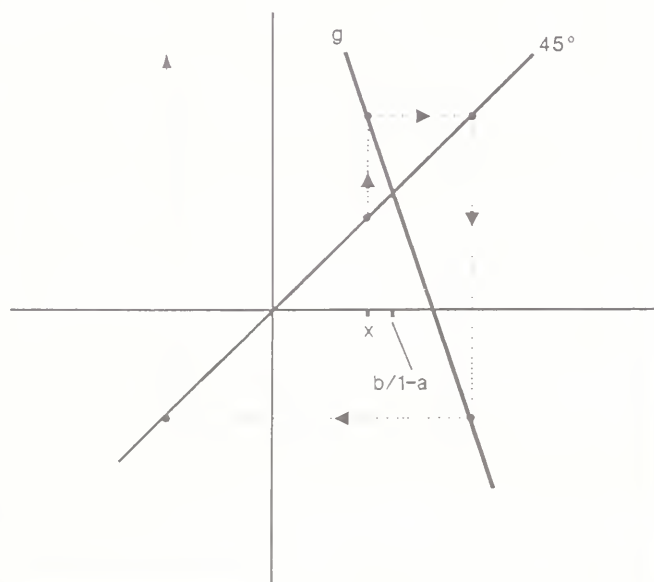
Orbit diagram for linear function ( $a < -1$ )

Figure 2(b)

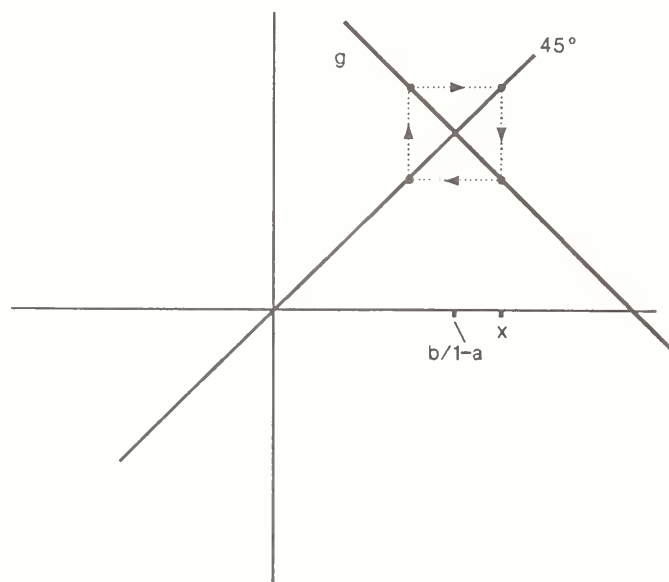
Orbit diagram for linear function ( $a = -1$ )

Figure 2(c)

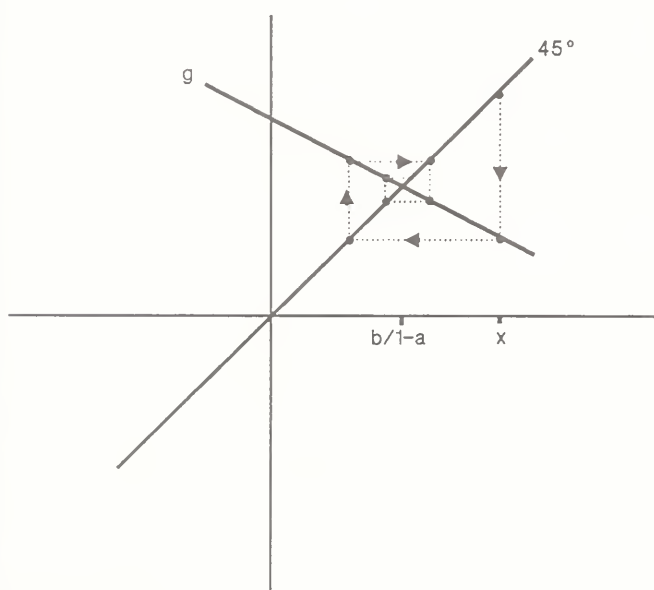
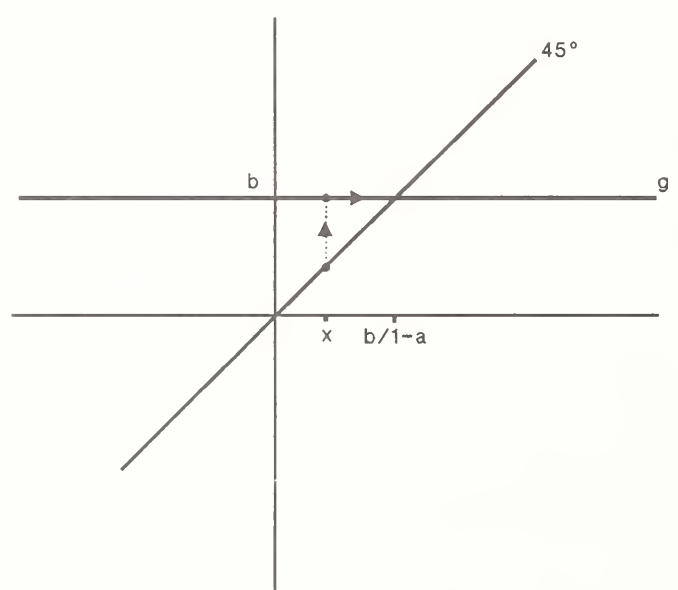
Orbit diagram for linear function ( $-1 < a < 0$ )

Figure 2(d)

Orbit diagram for linear function ( $a = 0$ )

general. To establish it, suppose a continuous function  $f$  has a convergent orbit  $x, f(x), f^2(x), \dots, f^n(x), \dots$ . Let  $L$  be the limit. Then:

$$\begin{aligned} f(L) &= f\left(\lim_{n \rightarrow \infty} f^n(x)\right) \\ &= \lim_{n \rightarrow \infty} f^{n+1}(x) \\ &= L, \end{aligned}$$

so that  $L$  is a fixed point of  $f$ . Thus, in partial answer to our guiding question, "where does the system go," we can reply: if it converges to any finite limit, that

limit must be a fixed point. Correspondingly, if an economy converges to an equilibrium, the equilibrium state must be a fixed point of the system function.

Closely related to fixed points are points whose orbits may leave but later return (see fig. 2(b)). A point  $x$  is called a *periodic point* of  $f$  with *period*  $n$  if  $f^n(x) = x$ . The smallest positive  $n$  for which the latter equation holds is called the *prime period* of  $x$ . It can be shown that any period of  $x$  is a multiple of the prime period.

Every fixed point of a function  $f$  is a periodic point of  $f$  of prime period 1. It is also true that every periodic point is a fixed point (though not of the same function), since the periodicity condition  $f^n(x) = x$  is nothing but

Figure 2(e)

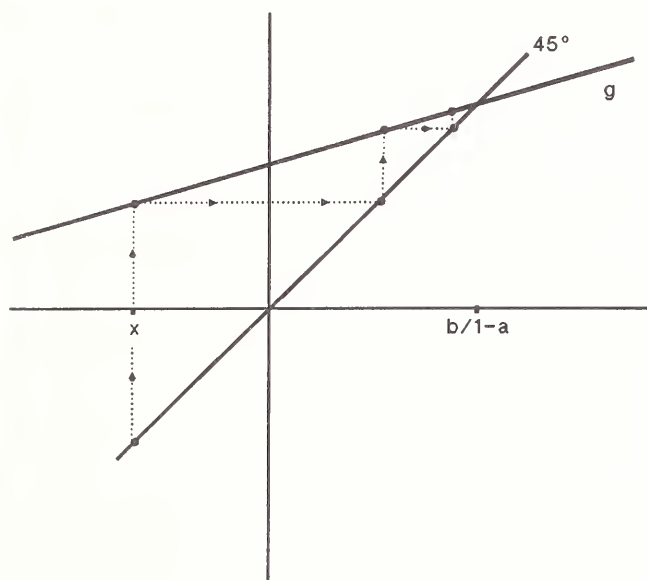
Orbit diagram for linear function ( $0 < a < 1$ )

Figure 2(f)

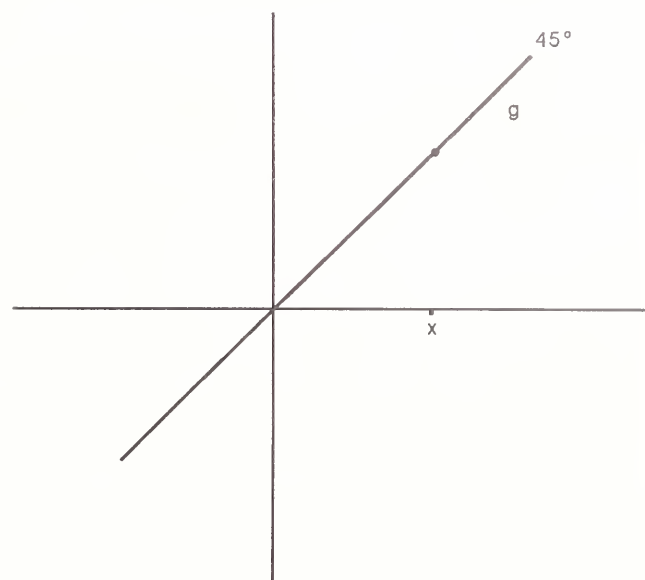
Orbit diagram for linear function ( $a=1, b=0$ )

Figure 2(g)

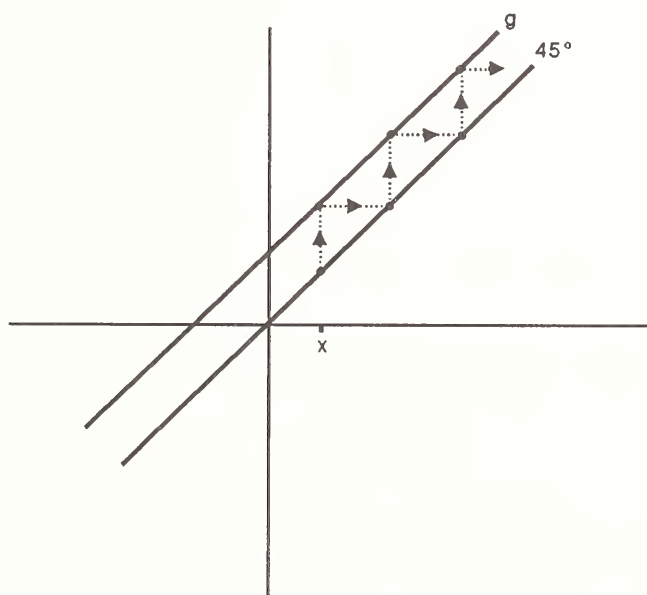
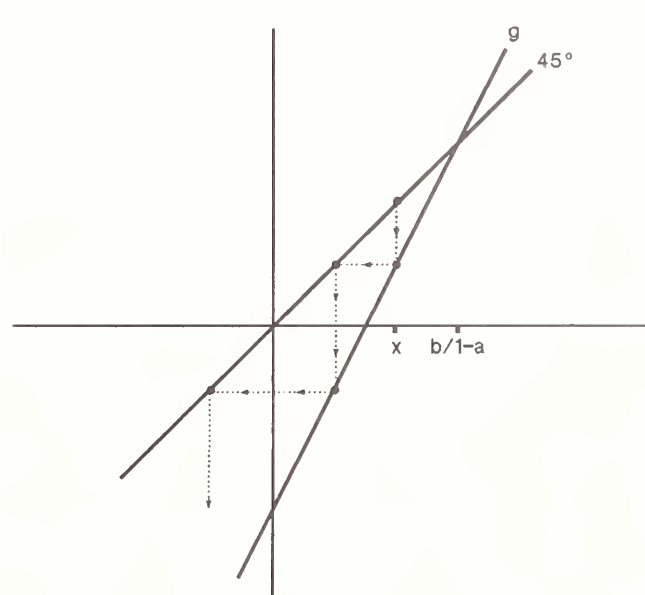
Orbit diagram for linear function ( $a=1, b \neq 0$ )

Figure 2(h)

Orbit diagram for linear function ( $a > 1$ )

the assertion that  $x$  is a fixed point of the function  $f^n$ . Thus, properties of fixed points have counterparts for periodic points, and vice versa.

If  $x$  is a periodic point of  $f$  having prime period  $n_0$ , then necessarily,

$$f^{n_0+1}(x) = f(f^{n_0}(x)) = f(x),$$

$$f^{n_0+2}(x) = f(f^{n_0+1}(x)) = f^2(x),$$

and so on. It follows that the orbit of  $x$  reduces to a finite set consisting of the distinct points  $x, f(x), f^2(x), \dots, f^{n_0-1}(x)$ , through which the system endlessly

cycles. (Such an orbit is called a *cycle of length  $n_0$* .) As a consequence, whenever a system with a computationally tractable law of motion is initialized at a point known to have a small period, the system's entire future evolution can, as a practical matter, be calculated.

In recent decades, there have been some remarkable discoveries concerning when the existence of a cycle of one length implies the existence of cycles of other lengths. Li and Yorke (10) show that if  $f$  is any continuous function mapping an interval  $J$  into itself, and if some point in  $J$  is periodic for  $f$  with prime period 3, then, for every positive integer  $n$ , there is a periodic



point in  $J$  having prime period  $n$ .<sup>1</sup> In brief: if there is a cycle of length 3, there must be cycles of all lengths.

The Li-Yorke Theorem is actually only a part of a more general result of Sarkovskii (see (6)) that may be described as follows. List the entire set of positive integers in the following manner:

$$\begin{array}{l} 3, 5, 7, \dots \\ 2 \cdot 3, 2 \cdot 5, 2 \cdot 7, \dots \\ 2^2 \cdot 3, 2^2 \cdot 5, 2^2 \cdot 7, \dots \\ 2^3 \cdot 3, 2^3 \cdot 5, 2^3 \cdot 7, \dots \\ \vdots \\ 2^n \cdot 3, 2^n \cdot 5, 2^n \cdot 7, \dots \\ \vdots \\ \vdots \\ \dots 2^3, 2^2, 2, 1. \end{array}$$

This list is called the “Sarkovskii ordering” of the positive integers. Observe that the odd integers exceeding 1 are listed first, followed by the various positive powers of 2 times the odd integers exceeding 1, followed finally by the pure powers of 2 in reverse order. Now, suppose  $f$  is a continuous function mapping the number line into itself. Then, Sarkovskii’s Theorem states that, if  $f$  has a periodic point of prime period  $k$  and  $k'$  is any integer appearing later in the list than  $k$ , then  $f$  also has a periodic point of prime period  $k'$ . One consequence of this result is that if  $f$  has any cycle whose length is not a pure power of 2, then  $f$  must have cycles of infinitely many different lengths. Thus, for example, if an annual iterative economic model with one endogenous variable exhibits a business cycle of length 5 years, then (for other initial points) the model must be capable of generating business cycles of infinitely many other lengths. While Sarkovskii’s Theorem pertains only to functions of one variable and thus would be directly applicable to, at most, a limited class of dynamic economic models, it does serve to illustrate the thesis that nonlinear dynamical systems are liable to impose unobvious but empirically relevant mathematical restrictions on economic behavior.

### Hyperbolic Points

Examination of the linear system reveals that, whenever  $|a| < 1$ , all orbits converge to the fixed point  $b/(1-a)$ . However, it is clear from figures 2(c), 2(d),

and 2(e) that, if a curve (that is, a nonlinearity) were introduced into the graph of the function  $g$  at some distance from  $b/(1-a)$ , any orbit originating near enough to  $b/(1-a)$  would still converge to  $b/(1-a)$ . Such local convergence does not depend on the slope of the graph of the function far away from the fixed point; what matters is only the slope, that is, the derivative, in a neighborhood of the fixed point. In fact, less obviously, but as we shall see momentarily, it is really only the derivative at the fixed point itself that matters.

Similarly, all orbits in the linear system originating elsewhere than  $b/(1-a)$  move away from  $b/(1-a)$  whenever  $|a| > 1$ . If a nonlinearity were introduced into the graph of  $g$  at a distance from  $b/(1-a)$ , any orbit originating sufficiently close to (but not precisely at)  $b/(1-a)$  would still move away from  $b/(1-a)$ , at least initially (the possibility of an eventual return is another issue). Again, for such local “aversion” to  $b/(1-a)$ , it turns out that only the derivative at  $b/(1-a)$  itself matters.

These observations lead to the following definitions. A fixed point  $p$  of a function  $f$  is called *hyperbolic* if  $|f'(p)| \neq 1$ . When  $|f'(p)| < 1$ ,  $p$  is called *attracting*, while when  $|f'(p)| > 1$ ,  $p$  is called *repelling*. These adjectives are justified by the following two propositions, which are readily established: (1) If  $p$  is an attracting hyperbolic fixed point, there is an interval containing  $p$  such that any orbit originating therein converges to  $p$ . (2) If  $p$  is a repelling hyperbolic fixed point, there is an interval containing  $p$  such that any orbit originating therein (but not at  $p$  itself) eventually leaves the interval (at least temporarily). For the function shown in figure 1, 0 is attracting hyperbolic while the other two fixed points are repelling hyperbolic.

In the literature, a periodic point  $x$  of  $f$  of prime period  $n$  is defined as hyperbolic if  $|(f^n)'(x)| \neq 1$ . The meaning of this definition becomes transparent once it is recalled that  $x$  is a fixed point of  $f^n$ .

In higher dimensional systems, the notion of the derivative at a point is expressed in terms of a Jacobian matrix, and a periodic point is defined as hyperbolic if none of the eigenvalues of this matrix has complex modulus one (that is, if none lies on the unit circle in the complex plane).

When a fixed point  $p$  is hyperbolic attracting, the system can be considered stable at  $p$  with respect to changes in initial conditions. If the system is initialized at  $p$ , it will, of course, remain there. More important, though, the system will converge to  $p$  even if it is not initialized there, as long as it is initialized sufficiently near  $p$ .

In the same vein, a hyperbolic repelling fixed point  $p$  can be considered a point of instability of the system with respect to changes in initial conditions. While the system will remain at  $p$  if initialized precisely there, it

<sup>1</sup>Italicized numbers in parentheses cite sources listed in the References section at the end of this article.

will move away from  $p$  whenever it is initialized sufficiently close to, but not at,  $p$ .

## Structural Stability

The stability property enjoyed by an attracting hyperbolic point concerns the effect of a slight change in the initial condition; the underlying model, however, remains fixed. We now discuss another notion of stability, structural stability, which concerns the effect of a slight change in the model itself.

In essence, a model is structurally stable if small changes in the model's structure leave dynamical behavior qualitatively unchanged. To understand why this property is important for empirical work, suppose that, from a collection of economic models sharing the same functional form and differing only in their values of some structural parameter vector  $w$ , we were to attempt to select the model  $M_{w_0}$  that truly described reality. Suppose further, though, that there existed parameter vectors  $w$  arbitrarily close to  $w_0$  whose corresponding models  $M_w$  had dynamical behavior differing from that of  $M_{w_0}$ . Then, as a practical matter, we could never confidently determine the true economic dynamics of the situation, for even the slightest error in econometrically estimating  $w_0$  (such as due to computer rounding) would leave us vulnerable to having arrived at a dynamically inequivalent  $M_w$ . What we would prefer is for our parameterized collection of models to satisfy the condition that, whenever  $w$  is sufficiently close to  $w_0$ ,  $M_w$  must be dynamically equivalent to  $M_{w_0}$ . This property, structural stability, is probably implicitly assumed by most economists engaged in computer modeling of dynamic economic systems. However, as we shall soon see, even the simplest nonlinear systems can be structurally unstable. Thus, structural stability cannot be taken for granted.

To make these ideas more concrete, let us consider the meaning of structural stability for discrete one-dimensional dynamical systems. We first need to clarify what could be meant in this context by "a small change in the model's structure."

Since these systems are entirely determined by the function being iterated, it makes sense to interpret a small change in the system as meaning a small change in the underlying function. But, to change a function slightly really means to introduce a new function that is, in some sense, near the original. How, then, can we measure the "nearness" of two functions? In the theory of structural stability, the following method has proved effective. Suppose  $f$  and  $g$  are  $r$ -times differentiable functions defined on an interval  $J$ . Usually, one assumes also that  $f^{(r)}$  and  $g^{(r)}$  are continuous, so that  $f$  and  $g$  are  $r$ -times continuously differentiable and thus

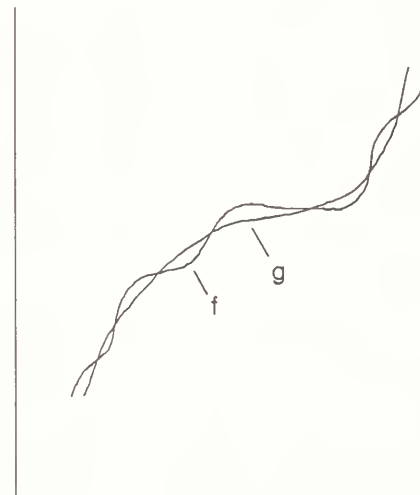
belong to the set  $C^r$  of all such functions.<sup>2</sup> Define the " $C^r$ -distance" between  $f$  and  $g$  as

$$d_r(f,g) = \sup_x \max \{ |f(x)-g(x)|, |f'(x)-g'(x)|, \dots, |f^{(r)}(x)-g^{(r)}(x)| \},$$

where the supremum<sup>3</sup> is taken over all  $x$  in  $J$ . (For each  $x$  in  $J$ , there is a corresponding maximum of absolute values as shown; the supremum is over this set of maxima.) Then,  $f$  and  $g$  are considered " $C^r$ -close" when  $d_r(f,g)$  is small, that is, when  $f$  is pointwise close to  $g$  and the first  $r$  derivatives of  $f$  are pointwise close to those of  $g$ . Figure 3 shows two functions that are  $C^0$ -close but not  $C^1$ -close.

Our next step in making precise the notion of structural stability is to clarify what is meant by the dynamical "equivalence" of two dynamical systems. Toward this end, suppose  $f$  and  $g$  are continuous functions mapping an interval  $J$  into itself. By a *homeomorphism*<sup>4</sup> of  $J$  we mean a continuous invertible function mapping  $J$  onto itself. Thus, a homeomorphism of  $J$  is a one-to-one correspondence between  $J$  and itself such that nearby points are sent to nearby points. Since the inverse function of a homeomorphism of  $J$  is itself necessarily continuous, this "preservation of nearness" operates in both directions. As an example, the function  $h$  defined for all  $x$  in the interval

Figure 3  
Functions  $C^0$ -close but not  $C^1$ -close



<sup>2</sup>By convention, when  $r = 0$ ,  $f^{(r)} = f$ ; that is, the 0th derivative of a function is the function itself. Since  $C^0$  is defined as the set of all functions with continuous 0th derivative,  $C^0$  is simply the set of all continuous functions.

<sup>3</sup>The supremum of a set of numbers, denoted "sup," is the smallest upper bound of the set. Thus, for example, the supremum of the open interval  $(0,1)$  is 1. Supremum plays the role of maximum for sets that may not have a largest element. The supremum of a set with no finite upper bound is  $\infty$ .

<sup>4</sup>From the Greek *homeo-* (similar) + *morphism* (form).



$[-1,1]$  by  $h(x) = x^3$  is a homeomorphism of this interval.

We say  $f$  and  $g$  are *topologically conjugate* if there exists a homeomorphism of  $J$  such that, for each  $x$  in  $J$ ,

$$h(f(x)) = g(h(x)).$$

What this condition expresses is that, whenever  $f$  sends a point  $x$  to  $f(x)$ ,  $g$  sends the point corresponding to  $x$  (namely  $h(x)$ ) to the point corresponding to  $f(x)$  (namely  $h(f(x))$ ). Thus, the behavior of  $g$  corresponds in a continuous one-to-one manner to the behavior of  $f$ .<sup>5</sup>

When two functions are topologically conjugate, each precisely replicates the dynamical properties of the other, and the dynamical systems they generate may be considered equivalent. To illustrate this point, suppose  $f$  and  $g$  are topologically conjugate by means of a homeomorphism  $h$ . Also, suppose  $f$  has an orbit originating at  $x$  and converging to  $p$ . Then:

$$\begin{aligned} h(p) &= h\left(\lim_{n \rightarrow \infty} f^n(x)\right) \\ &= \lim_{n \rightarrow \infty} h(f^n(x)) \\ &= \lim_{n \rightarrow \infty} h(f(f^{n-1}(x))) \\ &= \lim_{n \rightarrow \infty} g(h(f^{n-1}(x))) \\ &\cdot \\ &\cdot \\ &= \lim_{n \rightarrow \infty} g^n(h(x)), \end{aligned}$$

that is, the orbit of  $g$  originating at  $h(x)$  converges to  $h(p)$ . Similarly, suppose  $y$  is a periodic point of  $f$  of period  $m$ . Then:

$$\begin{aligned} g^m(h(y)) &= g^{m-1}(g(h(y))) \\ &= g^{m-1}(h(f(y))) \\ &\cdot \\ &\cdot \\ &= h(f^m(y)) \\ &= h(y), \end{aligned}$$

so that  $h(y)$  is a periodic point of  $g$  of period  $m$ .

<sup>5</sup>Topology is the study of those properties a mathematical object retains when it is continuously transformed. The term "conjugate" originates in the Latin *com-* (together) + *jugum* (yoke) and literally means "joined or yoked together." Here, it is  $f$  and  $g$  that are "yoked together" by  $h$ .

A precise definition of structural stability is now within reach. Let  $f$  be an  $r$ -times continuously differentiable function mapping an interval  $J$  into itself. Then,  $f$  is called  *$C^r$ -structurally stable* if there exists an  $\epsilon > 0$  such that any  $r$ -times continuously differentiable function  $g$  that maps  $J$  into itself and satisfies  $d_r(f,g) < \epsilon$  is topologically conjugate to  $f$ .

While verifying structural stability can be difficult, examples of structural *instability* are not hard to find. Define  $f$  by:

$$f(x) = x - x^2,$$

and, for each  $\epsilon > 0$ , define  $g_\epsilon$  by:

$$g_\epsilon(x) = x - x^2 + \epsilon/2.$$

Note that, for each  $r$ , the functions  $f$  and  $g_\epsilon$  are  $r$ -times continuously differentiable and map the number line (here playing the role of the interval  $J$ ) into itself. A simple computation shows that, for every  $r$  and  $\epsilon$ ,  $d_r(f,g_\epsilon) < \epsilon$ . However, examination of the equations  $x - x^2 = x$  and  $x - x^2 + \epsilon/2 = x$  immediately establishes that  $f$  has only one fixed point while each  $g_\epsilon$  has two. Thus, for no  $\epsilon > 0$  can  $g_\epsilon$  be topologically conjugate to  $f$ . It follows that  $f$  cannot be  $C^r$ -structurally stable.

If there is a moral for agricultural economists in this discussion, it would seem to be that greater emphasis should be placed on confirming the structural stability of a dynamic model prior to its econometric estimation. In the absence of structural stability, estimation of a model would only single out one of a number of dynamically inequivalent approximations. It would therefore serve no clear purpose.

## An Example of Chaotic Dynamics

To gain a qualitative understanding of what is involved in chaotic dynamics, we now examine in detail the class of functions  $F_\mu$  ( $\mu > 1$ ) defined by:

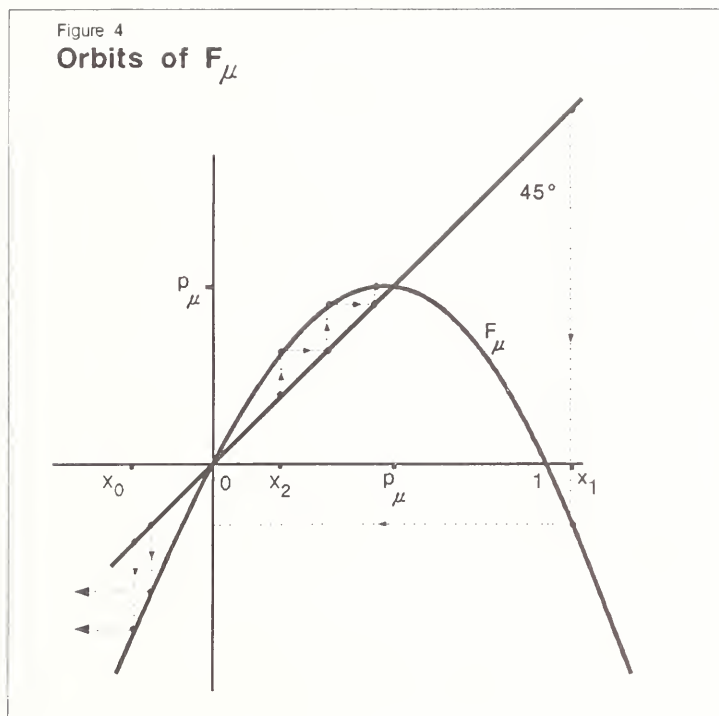
$$F_\mu(x) = \mu x(1-x).$$

Using functions from this class as the law of motion of a discrete dynamical system, I shall investigate the longrun behavior of all orbits, following the notation and approach of (6).

First, some basic facts (fig. 4). Let  $p_\mu = (\mu-1)/\mu$ . Then,  $0 < p_\mu < 1$ , and  $p_\mu$  is a fixed point of  $F_\mu$ . Another fixed point is 0. Since  $F_\mu(1) = 0$ , the orbit originating at 1 goes immediately to 0 and remains there. Finally, it is easy to show that any orbit of  $F_\mu$  originating at a point less than 0 (such as the point  $x_0$  of fig. 4) or greater than 1 (such as the point  $x_1$  of fig. 4) diverges to  $-\infty$ .



Figure 4  
Orbits of  $F_\mu$



Next, suppose  $1 < \mu < 3$ . Since  $F'_\mu(0) = \mu > 1$ , 0 is hyperbolic repelling. On the other hand, since  $F'_\mu(p_\mu) = 2 - \mu$  and  $-1 < 2 - \mu < 1$ ,  $p_\mu$  is hyperbolic attracting. One can show that the basin of attraction of  $p_\mu$  is precisely the open interval  $(0, 1)$ ; any orbit originating in this interval (such as at the point  $x_2$  of fig. 4) converges to  $p_\mu$ . We have thus determined the longrun behavior of all orbits of  $F_\mu$  for all values of  $\mu$  in the range  $1 < \mu < 3$ , and we have found nothing unusual in the dynamics arising in this parameter range.

However, as  $\mu$  increases beyond 3,  $F_\mu$  undergoes various qualitative changes. Among these is a change that occurs as  $\mu$  passes 4: the maximum value of  $F_\mu$  (namely  $F_\mu(1/2)$ , which equals  $\mu/4$ ) increases beyond 1, and some points in  $[0, 1]$  are thus mapped outside of  $[0, 1]$  by  $F_\mu$ . For any such point  $x$ , we have  $F_\mu(x) > 1$ , and it follows from a previous remark that the orbit of  $F_\mu(x)$ ,

$$F_\mu(x), F_\mu(F_\mu(x)), F_\mu^2(F_\mu(x)), \dots, F_\mu^n(F_\mu(x)), \dots,$$

that is,

$$F_\mu(x), F_\mu^2(x), F_\mu^3(x), \dots, F_\mu^{n+1}(x), \dots,$$

must diverge to  $-\infty$ . Hence, the orbit of  $x$  itself must diverge to  $-\infty$ . More generally, any orbit that originates in  $[0, 1]$  but does not remain in  $[0, 1]$  must diverge to  $-\infty$ .

Of particular interest is the parameter range  $\mu > 2 + \sqrt{5}$ . Although there are smaller values of  $\mu$  for which chaotic dynamics appears, Devaney (6) has shown that, when  $\mu > 2 + \sqrt{5}$ , the demonstration of chaotic dynamics can be accomplished relatively simply. We

assume, therefore, that  $\mu > 2 + \sqrt{5}$ , and will eventually find that on the set of initial points in  $[0, 1]$  whose orbits never leave  $[0, 1]$ ,  $F_\mu$  behaves chaotically.

Let  $\Lambda$  be this set. That is, let  $\Lambda$  be the set of all  $x$  in  $[0, 1]$  for which each term of the orbit of  $x$ ,

$$x, F_\mu(x), F_\mu^2(x), \dots, F_\mu^n(x), \dots,$$

is in  $[0, 1]$ . The first task is determining the structure of  $\Lambda$ , which will be done by ascertaining the structure of the complement of  $\Lambda$ , the set of those points of  $[0, 1]$  that are not in  $\Lambda$ .

For each  $n = 0, 1, 2, 3, \dots$ , let  $A_n$  be the set of all  $x$  in  $[0, 1]$  whose first  $n + 1$  orbit terms,

$$x, \dots, F_\mu^n(x),$$

are in  $[0, 1]$  but whose next orbit term,  $F_\mu^{n+1}(x)$ , is not. Observe that  $\Lambda$  consists precisely of those points of  $[0, 1]$  that lie in none of the  $A_n$ 's. Moreover, the  $A_n$ 's are pairwise disjoint. Thus, one can imagine constructing  $\Lambda$  through the following recursive process: from the interval  $[0, 1]$ , first remove the subset  $A_0$ ; next, from what remains, remove  $A_1$ , and so on. In general, when  $A_0, A_1, \dots, A_n$  have been removed from  $[0, 1]$ ,  $A_{n+1}$  must still (by disjointness) lie intact in the remaining subset of  $[0, 1]$ . Remove  $A_n$ , and continue this process *ad infinitum*. When all of the  $A_n$ 's have been removed from  $[0, 1]$ , the subset of  $[0, 1]$  that remains will be precisely  $\Lambda$ .

To picture what this process actually looks like, we rely on the fact that a point  $x$  lies in  $A_{n+1}$  if and only if  $F_\mu(x)$  lies in  $A_n$ . (This property follows from the definition of the  $A_n$ 's. In mathematical parlance,  $A_{n+1}$  is the pre-image of  $A_n$  relative to the function  $F_\mu$ .) Now,  $A_0$  is clearly an open interval of length less than 1 centered at  $1/2$ . Thus, removing  $A_0$  from  $[0, 1]$  leaves two disjoint closed intervals,  $B_0^1$  and  $B_0^2$  (fig. 5). To construct  $A_1$ , visualize a copy of  $A_0$  on the  $y$ -axis by reflecting  $A_0$  around the  $45^\circ$  line (fig. 6). Then, determine from the graph of  $F_\mu$  what points on the  $x$ -axis are mapped by  $F_\mu$  into  $A_0$ . The set of all such points will be  $A_1$  (fig. 7). Note that, since the graph of  $F_\mu$  rises continuously from 0 beyond 1 and then (farther to the right) descends continuously from beyond 1 back down to 0,  $A_1$  consists of two disjoint open intervals, each lying inside of (and at a positive distance from the endpoints of) one of the closed intervals  $B_0^1, B_0^2$ . Thus, removing both  $A_0$  and  $A_1$  from  $[0, 1]$  leaves behind *four* disjoint closed intervals.

$A_2$  is constructed similarly. Visualize a copy of  $A_1$  on the  $y$ -axis and determine the set of all points on the  $x$ -axis that are mapped by  $F_\mu$  into  $A_1$ ; that set will be  $A_2$ , and it will consist of four disjoint open intervals, each lying inside of (and at a positive distance from the endpoints of) one of the four closed intervals left behind after the removals of  $A_0$  and  $A_1$  from  $[0, 1]$ .

Figure 5

Removing the interval  $A_0$  from  $[0,1]$

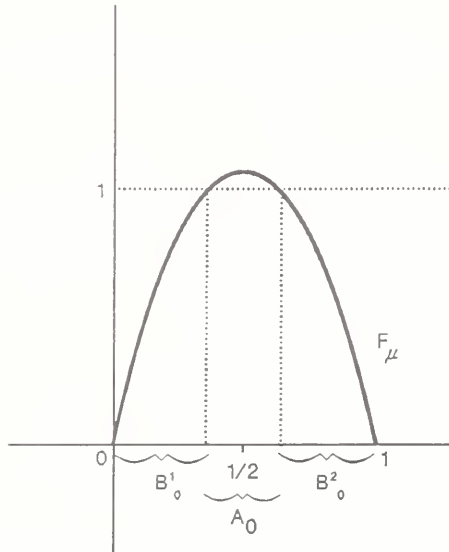


Figure 6

Copying the interval  $A_0$  onto the y-axis

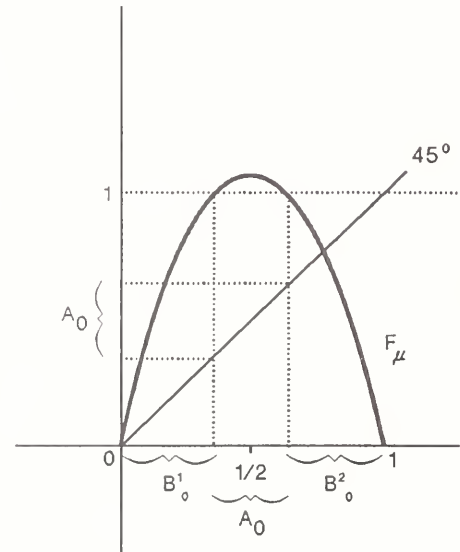
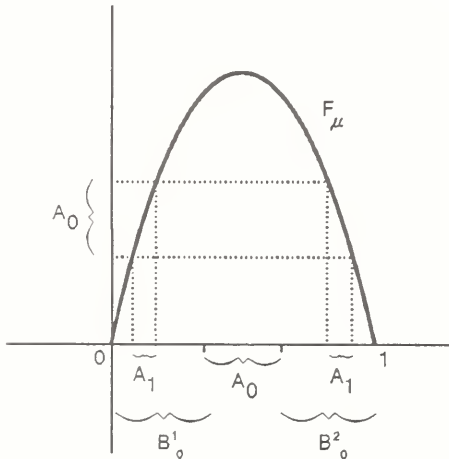


Figure 7

Constructing the set  $A_1$



This process can be continued. In general,  $A_n$  will consist of  $2^n$  disjoint open intervals, each lying inside of, and at a positive distance from the endpoints of, one of the  $2^n$  closed intervals left behind after the removals of  $A_0, \dots, A_{n-1}$ .

Thus, in brief,  $\Lambda$  is constructed by removing an open interval from the middle of the closed interval  $[0,1]$ , then removing open intervals from the middles of the remaining closed intervals, and so on, *ad infinitum*. This construction bears a striking resemblance to the

construction of a classic mathematical object called the *Cantor set*, a set defined by removing from  $[0,1]$  the open “middle third” interval  $(1/3, 2/3)$ , then removing the open middle third intervals  $(1/9, 2/9)$ ,  $(7/9, 8/9)$  from the two closed intervals remaining, and so on *ad infinitum*, always removing the open middle third interval of each closed interval remaining after the previous removals. The Cantor set has long been celebrated in mathematics for satisfying the following two conditions: (1) Its “length” is 0. (Indeed, by the formula for the sum of a geometric series, the total length of the disjoint intervals removed from  $[0,1]$  in constructing the Cantor set is:

$$\begin{aligned} 1 \cdot (1/3) + 2 \cdot (1/9) + 4 \cdot (1/27) + \dots &= \sum_{n=1}^{\infty} 2^{n-1}/3^n \\ &= (1/2) \sum_{n=1}^{\infty} (2/3)^n \\ &= 1. \end{aligned}$$

Modified forms of the Cantor set having positive length can be constructed by removing shorter intervals.) Yet, (2) the Cantor set contains as many points as all of  $[0,1]$ . (Specifically, it can be shown that there exists a one-to-one correspondence between the Cantor set and  $[0,1]$ . Since the elements of both sets can thus be paired off, the total number of points in each set must be the same. The fact that this number happens to be infinite should not be held against it. Infinite sets have sizes too.)

$\Lambda$  is known to share property (2) with the Cantor set. However, it also shares two further properties having more direct empirical implications: first,  $\Lambda$  is *perfect*, whose significance here is that, as near as desired to



any point of  $\Lambda$ , one can always find another point of  $\Lambda$ . That is, no point of  $\Lambda$  is isolated. Second,  $\Lambda$  is *totally disconnected* (it contains no open intervals),<sup>6</sup> from which it follows that, as near as desired to any point of  $\Lambda$ , one can always find a point of  $[0,1]$  that is *not* in  $\Lambda$ . As a consequence, whenever the dynamical system generated by  $F_\mu$  is initialized on  $\Lambda$ , its longrun behavior is “infinitely sensitive” to errors in the initial condition, since within any interval (no matter how small) around an intended starting point in  $\Lambda$ , there exist both points in  $\Lambda$  (whose orbits, by definition, remain in  $[0,1]$ ) and points not in  $\Lambda$  (whose orbits diverge to  $-\infty$ ). Thus, if one attempted to study this dynamical system on a computer, inevitable rounding errors in determining the points of  $\Lambda$  would make accurate simulation over  $\Lambda$  impossible.

This sensitivity of orbits to the initial condition, while *suggestive* of true “sensitive dependence on initial conditions,” must be carefully distinguished from it. The sensitivity just described compares orbits originating in  $\Lambda$  with orbits originating outside  $\Lambda$ . In contrast, true sensitive dependence refers to a kind of separating behavior between orbits originating nearby within the same set. A more formal definition will be provided later in this article.

Irregular sets such as  $\Lambda$  and the Cantor set have recently come to be referred to as “fractals” (from the Latin *fractus*, meaning “broken” and reflecting the disconnected character of such sets). Though the scientific community has not yet arrived at a consistent usage of this term, one often sees the following individual or joint criteria: exhibiting a high degree of jaggedness; self-similar (that is, defined by a recursive process in such a way that any part of the set, when magnified, looks the same as the entire set); and having noninteger dimension. (There are many ways to extend the usual concept of dimension (0 for a point, 1 for a curve, 2 for a surface, and so on) to more complicated sets. *Hausdorff* dimension (13), perhaps the most widely used, assigns to the Cantor set a dimension of  $\ln 2 / \ln 3$ , or approximately 0.63. Some other notions of dimension suggested for application to fractal sets are *information* dimension, *correlation* dimension, and *Lyapunov* dimension (see (17)).)

Until relatively recently, the Cantor-like sets now called fractals were considered exotic structures belonging solely to the world of pure mathematics. The discovery of their intimate connection with nonlinear dynamics has been striking. However, they are now understood to be a typical concomitant of nonlinear dynamical systems. (See, for example, (9, 12, 17) and the references contained therein.) They have been detected in the form of attractors (a fractal

attractor is called a *strange* attractor) and in the form of the boundary between competing basins of attraction. (Consider an economic model that allows different initial conditions to generate different equilibria. Here, the boundary between basins of attraction corresponding to distinct equilibria may be a fractal exhibiting a type of sensitivity to the initial condition noted earlier: the slightest movement away from an initial point lying in one basin of attraction may move the system to a new basin of attraction and thus cause it to evolve toward a new attractor. The equilibrium generated by an initial condition lying on this boundary would be unpredictable.) In addition, fractals can appear in the form of the state space region on which chaotic behavior is manifested (see the next section).

## Symbolic Dynamics

Having determined the structure of the set  $\Lambda$  of all points whose  $F_\mu$ -orbits remain in  $[0,1]$ , I now demonstrate the chaotic behavior of  $F_\mu$  on this set.

It turns out that if one attempts to analyze the orbits of  $F_\mu$  by direct computation the problem soon becomes prohibitively complex. Therefore, one constructs a *model* that abstracts from the phenomenon under study only its essential features. More specifically, I will construct a new dynamical system that is dynamically equivalent to the system determined by  $F_\mu$  on  $\Lambda$  but far easier to analyze. This approach, used commonly in the theory of dynamical systems, is called the method of *symbolic dynamics*.

The state space of our model dynamical system will be the set  $\Sigma_2$  of all sequences of 0's and 1's. We can represent a typical element of  $\Sigma_2$  as an infinite vector,

$$(s_0, s_1, s_2, s_3, \dots),$$

where  $s_i$ , either 0 or 1, is the  $i$ th term of the sequence. Note that we are starting our sequences with a “0th” term rather than a “1st” term. That is, our sequences are defined on the set of nonnegative integers rather than the set of positive integers. Later, this arrangement will enable us to associate the terms  $x$ ,  $F_\mu(x)$ ,  $F^2_\mu(x)$ ,  $F^3_\mu(x)$ , ... of an orbit of  $F_\mu$  with the terms  $s_0$ ,  $s_1$ ,  $s_2$ ,  $s_3$ , ... of a certain sequence in  $\Sigma_2$ .

An example of an element of  $\Sigma_2$  is the vector:

$$(0, 1, 0, 1, 0, 1, \dots),$$

(that is,  $(s_0, s_1, s_2, s_3, \dots)$ , where  $s_i = 0$  if  $i$  is even and  $s_i = 1$  if  $i$  is odd). Again, *any* sequence of 0's and 1's is allowed as an element of  $\Sigma_2$ .

The system function in our model—the function whose dynamics on  $\Sigma_2$  will parallel that of  $F_\mu$  on  $\Lambda$ —will be the

<sup>6</sup>This property should seem at least plausible in view of the method of construction of  $\Lambda$ . It is in proving this property that the assumption that  $\mu > 2 + \sqrt{5}$  is first put to use. See (6). Interestingly, the property implies that every point of  $\Lambda$  is on the boundary of  $\Lambda$ .



shift operator,  $\sigma$ , defined at any sequence  $(s_0, s_1, s_2, s_3, \dots)$  in  $\Sigma_2$  by:

$$\sigma \left( (s_0, s_1, s_2, s_3, \dots) \right) = (s_1, s_2, s_3, s_4, \dots).$$

Observe that  $\sigma$  maps a sequence to a new one whose  $i$ th term is the  $(i+1)$ st term of the original. (Note also that the initial term of the original sequence is ignored in forming the new one. Thus  $\sigma$ , like  $F_\mu$ , is not invertible.) Symbolically, we may write:

$$(\sigma(s))_i = s_{i+1},$$

for any sequence  $s$  in  $\Sigma_2$ .

Next, we make precise the meaning of the dynamical equivalence of  $\sigma$  and  $F_\mu$ . To do so, we generalize our earlier notion of dynamical equivalence, which relied on the concepts of homeomorphism and topological conjugacy. First, however, we must introduce a general definition of "continuous function."

Suppose  $f$  is a function mapping a set  $X$  into a set  $Y$ . Recall the intuitive meaning of continuity: as  $x$  approaches  $x'$ ,  $f(x)$  approaches  $f(x')$ . Suppose each of  $X$  and  $Y$  has been assigned some measure of the "distance" between its points. (Just as the sets  $X$  and  $Y$  can be quite different, these measures of distance can be quite different too.) Then, the intuitive meaning of continuity becomes: as the distance (in  $X$ ) between  $x$  and  $x'$  approaches 0, the distance (in  $Y$ ) between  $f(x)$  and  $f(x')$  approaches 0. More formally,  $f$  is continuous at  $x'$  if, for any  $\epsilon > 0$ , there exists a  $\delta > 0$  such that, whenever the distance in  $X$  between  $x$  and  $x'$  is less than  $\delta$ , the distance in  $Y$  between  $f(x)$  and  $f(x')$  is less than  $\epsilon$ .

The set  $\Lambda$  comes equipped with a natural measure of distance: the absolute value of the difference between two points. As for  $\Sigma_2$ , we now define the distance between any of its sequences  $s$  and  $t$  to be:

$$d(s, t) = \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{2^i}.$$

It is not difficult to prove that, with this distance measure,  $\sigma$  is continuous on  $\Sigma_2$ . We already know, of course, that  $F_\mu$  is continuous on  $\Lambda$ .

We are now able to generalize our earlier notions of homeomorphism, topological conjugacy, and (thus) dynamical equivalence to apply to  $\sigma$  and  $F_\mu$ . Suppose  $X$  and  $Y$  are any sets each of which has been assigned a distance measure. Then, by a *homeomorphism* between  $X$  and  $Y$  we mean an invertible function mapping  $X$  onto  $Y$  such that both the function and its inverse are continuous. Suppose  $f$  is a continuous function mapping  $X$  into itself and  $g$  is a continuous function mapping  $Y$  into itself. Then, we say  $f$  and  $g$  are

*topologically conjugate* if there exists a homeomorphism  $h$  between  $X$  and  $Y$  such that, for each  $x$  in  $X$ ,

$$h(f(x)) = g(h(x)).$$

As with our earlier definition, topologically conjugate functions map corresponding points to corresponding points, exhibit the same dynamical properties, and may be considered dynamically equivalent.

We now define a homeomorphism between  $\Lambda$  and  $\Sigma_2$  by means of which  $F_\mu$  and  $\sigma$  can be shown to be conjugate. Recall that, under our continuing assumption that  $\mu > 2 + \sqrt{5}$ , we earlier defined  $A_0$  as the set of points in  $[0, 1]$  whose  $F_\mu$ -values lie outside  $[0, 1]$ , and we defined  $B_0^1$  and  $B_0^2$  as the disjoint closed intervals remaining when  $A_0$  is removed from  $[0, 1]$  (see fig. 5). To facilitate our defining a homeomorphism, we rename the intervals  $B_0^1$  and  $B_0^2$  as " $I_0$ " and " $I_1$ ," respectively (fig. 8).

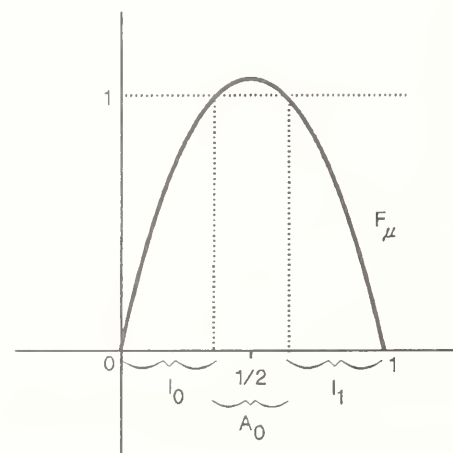
We define a function,  $S$ , from  $\Lambda$  to  $\Sigma_2$  as follows. Let  $x$  be an arbitrary point in  $\Lambda$ . By definition, the orbit of  $x$  never leaves  $[0, 1]$ ; thus, the orbit must remain within the sets  $I_0, I_1$ . We associate with  $x$  a sequence,  $S(x)$ , of 0's and 1's whose  $i$ th term ( $i = 0, 1, 2, 3, \dots$ ) is defined by:

$$(S(x))_i = \begin{cases} 0 & \text{if } F_\mu^i(x) \text{ lies in } I_0 \\ 1 & \text{if } F_\mu^i(x) \text{ lies in } I_1. \end{cases}$$

$S(x)$ , called the *itinerary* of  $x$ , is obviously an element of  $\Sigma_2$ . Thus,  $S$  is a function mapping  $\Lambda$  to  $\Sigma_2$ . Rather amazingly, it turns out that  $S$  is, in fact, a homeomorphism between  $\Lambda$  and  $\Sigma_2$ . Moreover, by means of  $S$ ,  $F_\mu$  and  $\sigma$  can be shown to be topologically conjugate. The

Figure 8

### The intervals $I_0$ and $I_1$



proofs of these facts go beyond the scope of this article (see (6)).

The conjugacy between  $F_\mu$  and  $\sigma$  establishes their dynamical equivalence and permits information gleaned from  $\sigma$  to be applied to  $F_\mu$ . To begin to exploit this feature, let us determine how many points of period  $n$   $F_\mu$  has. Now, by dynamical equivalence,  $\sigma$  must have exactly the same number of points of period  $n$  as  $F_\mu$ . However, for a sequence  $s$  in  $\Sigma_2$  to be of period  $n$  with respect to  $\sigma$  means that shifting  $s$   $n$  times produces  $s$  again, that is,  $s_{i+n} = s_i$  for each  $i$ . It follows that  $s$  must be a repeating sequence of the form:

$$(s_0, \dots, s_{n-1}; s_0, \dots, s_{n-1}; \dots).$$

There are precisely  $2^n$  ways of arranging 0's and 1's to form a finite string  $s_0, \dots, s_{n-1}$ ; hence,  $\sigma$  must have exactly  $2^n$  periodic points of period  $n$ . The same holds, then, for  $F_\mu$ . Using symbolic dynamics, we have thus established that, for example,  $F_\mu$  has 64 periodic points of period 6, or  $2^{117}$  periodic points of period 117.

A subset  $X'$  of a set  $X$  endowed with a distance measure is called *dense* in  $X$  if, for any point  $x$  in  $X$ , one can find some point from  $X'$  as close to  $x$  as desired. The set of all periodic points of  $\sigma$  is dense in  $\Sigma_2$ . In fact, given any sequence  $s$  in  $\Sigma_2$  and any  $\epsilon > 0$  (no matter how small), choose  $n$  so that  $1/2^n < \epsilon$ . Define a repeating, hence periodic, sequence  $s'$  in  $\Sigma_2$  by:

$$s' = (s_0, \dots, s_n; s_0, \dots, s_n; \dots).$$

(Note that  $s'$  merely repeats the first  $n + 1$  terms of  $s$ .) From the definition of our distance measure  $d(\cdot, \cdot)$  on  $\Sigma_2$ , it follows that:

$$\begin{aligned} d(s, s') &= \sum_{i=0}^n 0 + \sum_{i=n+1}^{\infty} \frac{|s_i - s'_i|}{2^i} \\ &\leq \sum_{i=n+1}^{\infty} 1/2^i \\ &= 1/2^n \\ &< \epsilon. \end{aligned}$$

Thus, the set of periodic points of  $\sigma$  is dense in  $\Sigma_2$ . Since dynamical equivalence is known to encompass denseness properties, we can conclude that the set of periodic points of  $F_\mu$  is dense in  $\Lambda$ .

The preceding result tells us that cyclic orbits can be found originating arbitrarily near any point of  $\Lambda$ . We next show that erratic orbits can also be found originating arbitrarily near any point of  $\Lambda$ .

Define a sequence  $s^*$  in  $\Sigma_2$  by concatenating all finite strings of 0's and 1's as follows. First list all strings of length 1 (there are only two of these, "0" and "1"), then list all strings of length 2 (there are four of these:

"0,0", "0,1", "1,0", and "1,1"), then all strings of length 3, and so on. In general, after all of the  $2^n$  strings of length  $n$  have been listed, continue with all strings of length  $n + 1$  and beyond. Thus:

$$s^* = (0, 1; 0, 0, 0, 1, 1, 0, 1, 1; 0, 0, 0, 0, 0, 1, \dots).$$

We shall show that the orbit of  $s^*$  with respect to  $\sigma$  is dense in  $\Sigma_2$ . In fact, given any sequence  $s$  in  $\Sigma_2$  and any  $\epsilon > 0$ , choose  $n$  so that  $1/2^n < \epsilon$ , and observe that the string  $s_0, \dots, s_n$  consisting of the first  $n + 1$  terms of  $s$  must appear somewhere in  $s^*$ . By the definition of  $\sigma$ , there must therefore exist a  $k$  such that:

$$\sigma^k(s^*) = (s_0, \dots, s_n, \dots),$$

that is, such that the first  $n + 1$  terms of  $\sigma^k(s^*)$  and  $s$  agree. As before, the formula defining our distance measure then implies that:

$$d(\sigma^k(s^*), s) \leq 1/2^n < \epsilon.$$

It follows that the orbit of  $s^*$  is dense in  $\Sigma_2$ . By the dynamical equivalence between  $F_\mu$  and  $\sigma$ , we can thus conclude that  $S^{-1}(s^*)$ , the point in  $\Lambda$  corresponding to  $s^*$  under the inverse of the itinerary homeomorphism  $S$ , has a dense  $F_\mu$ -orbit in  $\Lambda$ . For brevity, put  $x^* = S^{-1}(s^*)$ .

The fact that the orbit of  $x^*$  is dense in  $\Lambda$  implies that, for any  $x$  in  $\Lambda$  and any  $\epsilon > 0$ , there is an orbit point  $F_\mu^m(x^*)$  lying in the interval  $(x - \epsilon, x + \epsilon)$ . However, a simple argument shows that the orbit of  $F_\mu^m(x^*)$ ,

$$F_\mu^m(x^*), F_\mu^{m+1}(x^*), F_\mu^{m+2}(x^*), \dots,$$

must also be dense in  $\Lambda$ . Thus, arbitrarily near any point of  $\Lambda$  there originates a dense orbit. Such an orbit would appear erratic and essentially random, for it would endlessly "dance" around  $\Lambda$ , visiting and revisiting the vicinity of each point of  $\Lambda$  infinitely many times.

Recent findings by both economists (4) and mathematicians (14) have shed additional light on this seemingly random character of chaotic orbits. There is now evidence that chaotic behavior is indeed often legitimately stochastic in the sense that chaotic orbits may be realizations of a stochastic process defined on the state space. In this situation, the longrun behavior of an economic variable may be described by an endogenously generated longrun probability distribution. Day and Shafer (4) proved the existence of such distributions in a standard dynamic macroeconomic model and calculated the distributions numerically. Their work suggests that, under some conditions, longrun point forecasting in a nonlinear economic model should be replaced by calculation of a longrun probability distribution over the entire state space (for further explanation, see (19)).



Finally, we define the hallmark of chaos—sensitive dependence on initial conditions—and verify that  $F_\mu$  exhibits this property. Let  $X$  be any set endowed with a distance measure and  $f$  a function mapping  $X$  to itself. We say  $f$  exhibits *sensitive dependence on initial conditions* if there exists a  $\delta > 0$  with the following property: for any  $x$  in  $X$  and any  $\epsilon > 0$ , there is a point  $x'$  in  $X$  within a distance of  $\epsilon$  from  $x$  such that, for some  $n$ , the distance between  $f^n(x)$  and  $f^n(x')$  exceeds  $\delta$ . Heuristically, sensitive dependence means that there is a constant  $\delta > 0$  such that, arbitrarily close to any point of  $X$ , one can find another point of  $X$  whose orbit eventually diverges (even if only temporarily) from that of the given point by more than  $\delta$ . (Lyapunov exponents are sometimes used as a pragmatic measure of this divergence (17).) Although this discrepancy in orbits is required only to exceed  $\delta$ , not to be arbitrarily large in absolute terms, it should be noted that the *ratio* of this discrepancy to the distance between  $x$  and  $x'$  will become arbitrarily large when  $x'$  is chosen arbitrarily close to  $x$ . It is in this sense that sensitive dependence implies unpredictable longrun behavior for orbits originating arbitrarily near one another.

To show that  $F_\mu$  exhibits sensitive dependence on initial conditions, let  $\delta$  be any positive number less than the distance between the intervals  $I_0$  and  $I_1$ , that is, less than the length of  $A_0$  (see fig. 8). Choose any  $x$  in  $\Lambda$  and any  $\epsilon > 0$ . Since, as noted earlier, no point of  $\Lambda$  is isolated, there must exist a point  $x'$  in  $\Lambda$  distinct from  $x$  whose distance from  $x$  is less than  $\epsilon$ . However, since the itinerary mapping,  $S$ , is invertible, distinct points in  $\Lambda$  must have distinct itinerary sequences. Thus, for some  $n$ ,

$$(S(x))_n \neq (S(x'))_n,$$

that is, either  $F_\mu^n(x)$  is in  $I_0$  and  $F_\mu^n(x')$  is in  $I_1$  or vice versa. It follows at once that the distance between  $F_\mu^n(x)$  and  $F_\mu^n(x')$  exceeds  $\delta$ , which proves the result.

While the examination of chaotic behavior over a fractal set such as  $\Lambda$  can reveal important aspects of the subject, chaos should not be viewed as a phenomenon that appears only on unusual sets. One can show, for example, that the function  $F_4$  given by  $F_4(x) = 4x(1-x)$  is chaotic on the entire interval  $[0,1]$ . Nor is the existence of chaos overly sensitive to functional form. The chaotic dynamics we have described for  $F_\mu$  will also be exhibited by essentially any hill-shaped function with sufficiently large slope.

## Conclusions

Recent findings in the field of nonlinear dynamical systems warrant a rethinking of traditional attitudes toward economic dynamics. It is now known that erratic longrun behavior and various forms of sensitivity to initial conditions can arise in even the simplest nonlinear models. Unless there are sound reasons in

economic theory to believe that a given dynamic economic process is linear, the process must be viewed as at least potentially liable to the type of chaotic behavior described here.

Chaos theory suggests that the long-range prediction of nonlinear economic processes may be subject to the same basic mathematical limitations as long-range weather prediction. In both cases, future behavior may appear independent of the initial conditions that produced it.

It is difficult to study this subject without experiencing a certain humility concerning our ability to control nonlinear economic processes through policy intervention. Nonlinear systems can behave in a counterintuitive manner. The conditions under which we can properly use mathematical models to predict the long-run implications of policy actions need to be clarified.

The discoveries of recent years might seem to have revealed intrinsic mathematical limits to economic prediction. Yet, a deeper understanding of the limitations of longrun point prediction should ultimately enhance, not diminish, the accuracy and credibility of the information we provide. A further enhancement may derive from the replacement, in certain cases, of long-run point forecasts by forecasts based, in part, on endogenous longrun probability distributions.

We have attempted in this article to sketch some of the major themes of contemporary nonlinear dynamics. Many topics, however, had to be omitted. (For example, we have not discussed how the nonlinear dynamics literature might contribute to improvements in shortrun forecasting models such as models of stock market behavior. An anonymous referee suggests that a small percentage reduction of shortrun forecast errors may be possible. Such a reduction would be of particular interest to arbitrage traders.) Recommended further background reading would include (in order, 9, 6, 17, 12, 7, 15). For a sampling of the nascent economics literature on chaos, see (1, 2, 3, 4, 5, 8, 11, 16, 18).

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# The Changing Structure of the U.S. Flour Milling Industry

C.S. Kim, William Lin, and Mack N. Leath

**Abstract.** *What causes the structural changes, in terms of number and size of flour mills, in the U.S. flour milling industry, and will the U.S. wheat flour supply be adequate in the year 2000? Simulation results indicate that rising disposable income and declining wheat prices are the primary reasons for changes in the size distribution of larger flour mills, while automation in production and higher disposable income are important factors for smaller mills. This study also projects that domestic wheat flour supply will be sufficient to meet increasing domestic demand by the year 2000, even though the number of U.S. wheat flour mills is projected to decline to 160 from the current 203.*

**Keywords.** *Markov chains, multinomial logit analysis, flour mills, hazard function, year 2000.*

The U.S. milling industry has experienced considerable structural change during the past two decades. Between 1973 and 1987, the number of plants milling hard-, soft-, and whole-wheat flour steadily declined to 211 from 279. The number of mills with daily capacity under 1,000 hundredweight (cwt) declined from 125 to 63 during the same period, while flour mills with daily capacity over 10,000 cwt increased from 24 to 42. At the same time, wheat flour production increased substantially from 255 million cwt to a record high 338 million cwt.

U.S. flour mills were typically built near wheat-producing areas prior to the 1950's when costs of shipping flour did not differ from that of shipping wheat. Since then, flour mills were more commonly built at metropolitan centers as the cost of shipping flour exceeded that of shipping wheat. Most companies built flour mills near population centers in the 1980's.

What are the causes of structural changes in the U.S. milling industry? Economists often have hypothesized that a major cause for expansion in size is to achieve economic efficiency in input use (1, 9, 21).<sup>1</sup> Under this assumption, expansion of size economies continues as long as additional investment reduces longrun average costs, indicating increasing returns to size. Others argue that expansion of size economies results from external pressures, such as changes in consumer behavior, wages, and disposable income (2). Under this assumption, the longrun average cost curve is

L-shaped, meaning that efficiency is constant over a broad range of output, and therefore, small mills compete effectively with larger mills. Consequently, the industry is not characterized by a concentration of larger mills at any particular output level. Some economists believe that domestic farm policies have significant impact on expansion of size economies (6), since these policies influence structural change in the U.S. milling industry.

Government subsidies to wheat producers include input subsidies, export subsidies, and price and income support policies associated with acreage reduction and/or conservation programs. Since members of the General Agreement on Tariffs and Trade (GATT) are currently negotiating for trade liberalization, the potential effects on domestic wheat flour supply should be examined.

This article investigates the causes of structural changes in terms of number and size of flour mills in the U.S. flour milling industry. Assumptions associated with stationary and nonstationary transition probabilities are tested by employing the Markov process and a multinomial logit model. We also assess the effects on structural changes within the milling industry due to technology changes, increases in domestic consumption due to changes in consumer taste and rise in disposable income, and domestic grain policies. Finally, we project the number of mills for each size category and wheat flour supply to the year 2000 under various scenarios, including trade liberalization in the world wheat market.

## A Markov Chain Analysis

The Markov chain model associated with stationary transition probabilities has been widely used to evaluate changes in the size distribution of firms (1, 8, 14, 15, 24). This model assumes that the observed movement of firms among specific size classes over specific time periods will continue until the industry reaches an equilibrium size distribution of firms. Therefore, expansion of size economies will continue as long as the additional investment will reduce longrun average cost, which coincides with increasing returns to size (4, 16).

To estimate meaningful transition probabilities, the maximum likelihood method can be used when time-ordered data that reflect intertemporal changes of firms over size categories are available. In some cases, however, when time-ordered movements of individual firms among size categories are not available, transition probabilities can be estimated with the probability-constrained quadratic programming (QP)

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<sup>1</sup>Italicized numbers in parentheses cite sources listed in the References section at the end of this article.



model, the probability-constrained minimum absolute deviation (MAD) model (15), or the probability-constrained minimization of median absolute deviation (MOMAD) model (13). Since the probability-constrained MOMAD model is considered to be superior to the probability-constrained QP model in estimating transition probabilities with limited aggregate time series data, and easier to use than the probability-constrained MAD model, the probability-constrained MOMAD model is used in this article.

Flour mills are grouped into four size classes based on the size of daily active capacity in hundredweight. Intervals used to define the four classes consist of  $0 \leq S_1 < 1,000$ ;  $1,000 \leq S_2 < 5,000$ ;  $5,000 \leq S_3 < 10,000$ ; and  $S_4 \geq 10,000$ . Under this selection of size classification, the exit of flour milling is treated in a manner analogous to the merging of independent operations (1). Indeed, within the past two decades, the ownership of most flour milling companies has changed mainly through acquisitions (9).

The transition matrix for the U.S. flour milling industry is estimated with data covering 1973-87. Proportion data for each size class are obtained from the *Milling Directory: Buyer's Guide* (20). The results obtained are:

$$M = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 \end{matrix} \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{matrix} & \begin{bmatrix} .9767 & .0099 & .0134 & .0000 \\ 0 & .8113 & .1590 & .0297 \\ 0 & .2011 & .7905 & .0084 \\ 0 & .0274 & 0 & .9726 \end{bmatrix} \end{matrix} \quad (1)$$

The estimated transition matrix (1) provides some useful information about the dynamic nature of size economies. The probabilities on the diagonal in matrix 1 indicate that most flour mills in size classes  $S_1$  and  $S_4$  are likely to remain at the same size from one period to another. However, approximately 20 percent of flour mills in class  $S_2$  will likely increase daily capacity, while 20 percent of flour mills in class  $S_3$  will likely decrease their daily capacity.

To project the proportions of mill sizes in year  $t$ , let  $W(0)$  be the initial row vector of proportions and  $W(t)$  be the proportion vector at time  $t$ . The conditional expectation of  $W(t)$  is given by:

$$W(t) = W(t-1) * M = W(0) * M^t, \quad (2)$$

where  $M$  is the transition matrix. Equation 2 is used to project the proportions for mill sizes to year 2000. Table 1 indicates that the proportion for the size class  $S_1$  will decline from 0.25 to 0.19, and the proportion for the size  $S_4$  will increase from 0.22 to 0.26, while the proportions for the sizes  $S_2$  and  $S_3$  will remain unchanged. Since the number of mills is expected to

decline even more in the near future, this increase in proportion for the size  $S_4$  may not generate enough supply of wheat flour to meet demand.

## A Multinomial Logit Analysis

One of the most demanding assumptions of the Markov chain model is that once the process of structural change has been established, the same process of change will continue until it reaches an equilibrium size distribution of firms. This stationary assumption of transition probabilities may not be attainable in some cases. The expansion of size economies may result from changes in exogenous variables, such as technology, wages, and consumer taste. Indeed, growing health concerns have contributed to the increase in per-capita wheat flour consumption by 13 percent during the period 1973-87, while population has grown by more than 15 percent (9). Even though real wages in the U.S. milling industry have been stable, total production worker hours have declined by 25 percent during the study period. More surprising, even though there have been structural changes among different size classes, no strong evidence exists of intrastructural changes in each size class (table 2). These exogenous changes may suggest that transition probabilities are nonstationary. In this section, the multinomial logit model developed by Parks (22) is modified and then applied to estimate selection probabilities of size categories.

Following McFadden (17), and Domencich and McFadden (7), the selection probability of the  $i$ th size category can be written as:

$$P_i = \exp(D_i) / \sum_{j=1}^m \exp(D_j), \quad i = 1, 2, \dots, m, \quad (3)$$

**Table 1—Projected proportions of mill sizes with a Markov process**

Year	Mill size			
	$S_1$	$S_2$	$S_3$	$S_4$
1990	0.2454	0.2900	0.2473	0.2173
1991	.2396	.2934	.2449	.2221
1992	.2340	.2958	.2434	.2268
1993	.2286	.2974	.2426	.2314
1994	.2233	.2987	.2421	.2359
1995	.2181	.2997	.2419	.2403
1996	.2130	.3005	.2418	.2447
1997	.2080	.3013	.2418	.2447
1998	.2032	.3019	.2418	.2531
1999	.1984	.3025	.2419	.2572
2000	.1938	.3031	.2420	.2611

**Table 2—Daily active average milling capacity of a flour mill (1977-89)**

Variable	Mill size			
	Size 1	Size 2	Size 3	Size 4
Mean (cwt)	997.15	2,604.35	6,734.53	14,376.31
Standard deviation	23.62	66.38	118.66	247.64
Variation of coefficient	.0237	.0255	.0176	.0172



where  $D_j$  is the estimated utility function  $U_j$  such that  $U_j = D_j + \epsilon_j$ , and all  $\epsilon_j$  are independently and identically distributed with a Weibull distribution. The normalized multinomial logit model is written as:

$$P_i = \exp(d_i) / [1 + \sum_{j=2}^m \exp(d_j)], \quad i = 2, 3, \dots, m, \quad (4)$$

$$P_1 = [1 + \sum_{j=2}^m \exp(d_j)]^{-1},$$

where  $P_i/P_1 = \exp(D_i - D_1) = \exp(d_i)$ .

The natural logarithm of the odds of choosing the  $i$ th size mill over the size class 1 in year  $t$  is written as:

$$\begin{aligned} \ln (P_{it}/P_{1t}) = & b_{i0} + b_{i1} X_{1t} + b_{i2} X_{2t} + \dots \\ & + b_{ik} X_{kt} + v_{it} \end{aligned} \quad (5)$$

$$i = 2, 3, \dots, m; t = 1, 2, \dots, T.$$

Since the selection probabilities  $P_{it}$  are unknown and are replaced by the observed relative frequencies  $p_{it}$ , Parks introduced an additional error term  $u_{it}$  such that equation 5 is now expressed as:

$$\begin{aligned} \ln (p_{it}/p_{1t}) = & b_{i0} + b_{i1} X_{1t} + \dots \\ & + b_{ik} X_{kt} + v_{it} + u_{it}, \end{aligned} \quad (6)$$

$$i = 2, 3, \dots, m; t = 1, 2, \dots, T,$$

where  $E(v_{it}) = 0$ ,  $E(u_{it}) = 0$ ,

$$\begin{aligned} E(v_{it}, v_{jt}) = & \sigma_{ij} \quad \text{for all } i \text{ and } j \\ = & 0 \quad \text{otherwise, and} \end{aligned}$$

$$E(u_{it}, u_{jt}) = \Omega$$

$$= 1/n_t \begin{bmatrix} 1/P_{1t} + 1/P_{2t} & 1/P_{1t} & \dots & 1/P_{1t} \\ 1/P_{1t} & 1/P_{1t} + 1/P_{3t} & \dots & 1/P_{1t} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1/P_{1t} & 1/P_{1t} & \dots & 1/P_{1t} + 1/P_{mt} \end{bmatrix}$$

Zellner and Lee showed that the joint estimation procedure produces more efficient estimators than do single-equation techniques (27). The joint multinomial logit equations associated with equation 6 can be written in compact notation as:

$$Y = X\beta + e, \quad (7)$$

where  $Y$  is an  $(m \times 1)/T$  vector,  $\beta$  is an  $(m \times k) \times 1$  vector such that  $\beta = (\beta_1', \beta_2', \dots, \beta_m')'$ ;  $e_t = v_t + u_t$ ;  $X = (X_1', X_2', \dots, X_T')'$ , where

$$X_t = \begin{bmatrix} X_{1t} & & & & \\ & X_{2t} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & X_{mt} \end{bmatrix},$$

and  $x_{it}$  is a  $(1 \times k)$  vector of explanatory variables.

The Parks-modified multinomial logit (MML) estimator is given by  $b_{MML} = (X'V^{-1}X)^{-1} X'V^{-1}Y$ , and the coefficient covariance matrix is estimated by  $(X'V^{-1}X)^{-1}$ , where  $V$  is a block diagonal matrix with  $(\Omega_t + \Sigma)$  in the  $t$ th

block,  $t = 1, 2, \dots, T$ , and  $\Sigma = \frac{1}{T} [S - \lambda \sum_{t=1}^T \Omega_t]$ , where  $S$  is a covariance matrix obtained from applying ordinary least squares to equation system 5. To explore explanatory variables in estimation of the selection probabilities, the price input-output model is given by (19):

$$P(i) * D(i) = \sum_j^n w_{ij} + Y_i \quad \text{for } i = 1, 2, \dots, n, \quad (8)$$

where  $D(i)$  is total production for sector  $i$ ,  $P(i)$  is price per unit of  $D(i)$ ,  $w_{ij}$  is dollar value of inputs bought by the  $j$ th production sector, and  $Y_i$  is the dollar value of final demand for sector  $i$  output. Equation 8 can be rewritten as equation 9 by dividing both sides of equation 8 by  $P(i)$ :

$$D(i) = \sum_j^n w_{ij}/P(i) + Y_i/P(i), \quad \text{for } i = 1, 2, \dots, n. \quad (9)$$

Equation 9 implies that total production of each sector can be explained with variables such as the ratio of input value to output price and the ratio of disposable income to output price. By inserting equation 9 into equation 3, the estimated model therefore includes independent variables for  $W^*H/P$  and  $Y^*N/P$ , where  $W$  is the hourly wage of production workers,  $H$  is total worker hours in millions of production workers,  $Y$  is disposable per capita income,  $N$  is population in millions, and  $P$  is the price of wheat flour per 100 pounds. Data for  $W$  and  $H$  are obtained from the *Census of Manufactures*,  $P$  is from (9), and  $Y$  and  $N$  are from the *Statistical Abstract of the United States*.

Table 3 shows the results of applying Parks's modified multinomial logit model. The coefficients of the estimated multinomial logit equation are not readily interpretable because the dependent variables are choice probabilities. Results show that the production workers' wage variable is statistically insignificant for the selection probability  $P_3$  for size class  $S_3$ , while all other variables are very significant for all selection probabilities. The value of  $R^2$  is 0.96, indicating that the structural changes of the U.S. milling industry

**Table 3—Estimates of the log odds of sizes  $S_2$ ,  $S_3$ , and  $S_4$  relative to  $S_1$**

Item	Dependent variable	Explanatory variables <sup>1</sup>		
		Constant	W*H/P	Y*N/P
$b_{MML}$	$\ln(P_2/P_1)$	-0.37804 (.13284)	-0.04371 (.01299)	0.00430 (.00076)
	$\ln(P_3/P_1)$	-.84638 (.14536)	-.02076 (.01413)	.00316 (.00083)
	$\ln(P_4/P_1)$	-1.62791 (.18846)	-.04508 (.01763)	.00695 (.00099)
$R^2 = .96$				
$S = \begin{bmatrix} .002070 & -.000923 & .000711 \\ -.000923 & .004031 & .003165 \\ .000711 & .003165 & .011623 \end{bmatrix} \hat{\Sigma} = \begin{bmatrix} .001010 & -.001357 & .000277 \\ -.001357 & .002807 & .002731 \\ .000277 & .002731 & .009975 \end{bmatrix}$				
$\lambda = .016586$				

<sup>1</sup>Estimated standard errors are in parentheses.

may be properly explained by variables W\*H/P and Y\*N/P. However, the reliability of estimates obtained from the Markov chain process should be compared with a multinomial logit analysis to project the size distribution of the U.S. milling industry.

### Simulation with Historical Data

Simulation with historical data is a simple way of verifying the reliability of estimated selection probabilities. We combined Theil's U-coefficient with historical data to investigate the effectiveness and accuracy of the selection probabilities estimated with the Markov chain process and the multinomial logit analysis (3, 26). The U-coefficient is given by:

$$U = \{\Sigma(P_i - A_i)^2/N\}^{1/2} / \{[\Sigma(A_i)^2/N]^{1/2} + [\Sigma(P_i)^2/N]^{1/2}\}, \quad (10)$$

where  $P_i$  is a simulated value,  $A_i$  is an actual value, and  $N$  is the number of observations. Theil's U-coefficient lies between zero and 1. If the U-coefficient is equal to zero, the simulated results are perfect, and if  $U$  equals 1, there is no relationship between the simulated and the actual values (3). Table 4 shows the U-coefficients for the selection probabilities. Results indicate that both models perform reasonably well, but the multinomial logit model displays better predictive efficiency than the Markov process model. Therefore, the modified multinomial logit model is the logical choice to investigate how explanatory variables affect the selection probabilities.

The estimated multinomial logit equations in table 3 can be presented as follows:

$$\ln(P_i/P_1) = a_i + b_i^*(W*H/P) + c_i^*(Y*N/P),$$

where  $i=2,3,4$ . (11)

The second term of the right-hand side of the equation,  $b_i^*(W*H/P)$ , represents the worker hour elasticity of the selection probabilities, and the third

**Table 4—Theil's U-coefficients for selection probabilities**

Item	Selection probabilities			
	$P_1$	$P_2$	$P_3$	$P_4$
Markov model	0.03984	0.02221	0.02594	0.03854
Multinomial logit model	.01247	.02051	.02617	.04447

term,  $c_i^*(Y*N/P)$ , represents the aggregate disposable income elasticity of the selection probabilities. The wheat flour price elasticity of the selection probabilities is represented by the negative sum of the worker hour and aggregate disposable income elasticity or, equivalently, by  $[a_i - \ln(P_i/P_1)]$ .

Elasticities are evaluated at the mean values. The estimated production worker hour elasticities of the selection probabilities  $[-0.9283, -0.4409, -0.9574]$  are for the size classes  $S_2$ ,  $S_3$ , and  $S_4$ , respectively. These results indicate that the selection probabilities for the size classes  $S_2$  and  $S_4$  increase proportionately to the reduction of production worker hours resulting from the mechanization in wheat flour processing. However, the selection probability for the size class  $S_3$  is expected to increase by half of the proportional reduction of production worker hours. The estimated aggregate disposable income elasticities of the selection probabilities are  $[0.9347, 0.6869, 1.5107]$  and the wheat flour price elasticities  $[-0.0064, -0.2460, -0.5533]$  are for the size classes  $S_2$ ,  $S_3$ , and  $S_4$ , respectively. The selection probabilities are very sensitive to changes in disposable income, while they are less sensitive to changes in wheat flour price.

Wheat flour price has declined by 53 percent from 1973 to 1987, while disposable income has risen by 32 percent. Exploring the causes of structural changes in the U.S. milling industry means using the total differentiation of equation 11:

$$dP_i = P_i^*\{(b_i^*W/P)*dH + (c_i^*/P)*d(Y*N)\}$$



$$-[(b_i * W * H + c_i * Y * N) / P^2] * (\delta P / \delta P_w) * \delta P_w\}, \quad (12)$$

where  $P_w$  is wheat price (cost) to produce 100 pounds of flour.<sup>2</sup>

Table 5 reveals changes in selection probabilities due to changes in production worker hours, aggregate disposable income, and wheat price. An increase in disposable income and a decrease in production worker hours have equally influenced the selection of the second size category,  $1,000 \leq S_2 < 5,000$  (table 5). However, the selections of the third size category,  $5,000 \leq S_3 < 10,000$ , and the fourth size category,  $S_4 \geq 10,000$ , have been influenced more by increases in disposable income and decreases in wheat price. Decreased production worker hours contributed only 25 percent of the changes in size categories  $S_3$  and  $S_4$ , while increased disposable income and decreased wheat price each account for approximately 37 percent of the changes in size classes  $S_3$  and  $S_4$ . These findings contradict previous studies that indicate the cause for expansion in size economies is to achieve economic efficiency in input use.

Wheat is one of the most protected crops in the United States. The government provides input subsidies, export subsidies, and price and income support policies associated with acreage reduction and/or conservation programs. As a result of government grain programs, per unit wheat costs for millers to produce 100 pounds of flour have declined by more than 70 percent from \$7.16 (1967 dollars) in 1974 to \$2.10 in 1988 (9). However, policymakers are discussing plans to phase out subsidies for grain producers, and members of GATT are negotiating for trade liberalization. The declining trend in wheat prices is expected to be affected by the removal of farm programs.

### Projecting the Number of Mills with a Hazard Function

Structural changes within the U.S. flour milling industry consist of both size distribution and the changing number of mills. In cases where a transition matrix is

**Table 5—Proportional contributions of production worker hours, disposable income, and wheat price to changes in selection probabilities  $P_2$ ,  $P_3$ , and  $P_4$**

Item	$P_2$	$P_3$	$P_4$
		<i>Percent</i>	
Production worker hours (H)	50.60	25.06	24.61
Disposable income ( $Y * V$ )	48.46	37.14	36.94
Wheat price ( $P_w$ )	.94	37.80	38.45

<sup>2</sup>The estimated marketing margin equation is:

$$P_f = .566047 + .876319 * P_w, \quad R^2 = .9953, \quad D.W. = 1.801, \quad n = 15, \\ (.071115) \quad (.016740),$$

where  $P_f$  is wheat flour price per 100 pounds and  $P_w$  is wheat price the millers paid to produce 100 pounds of flour.

made up of the conditions representing entry and exit of firms, the number of firms can be projected with the conditional expected value equation (3). For other cases, a simple regression method has been used to project mill numbers (8, 23). Even though this approach is simple to use, its specification suffers from the lack of a theoretical foundation.

Another approach is based on the so-called "reliability theory" (10, 11, 18). Time to failure, or life length,  $T$ , is defined as a continuous random variable following a Weibull probability density function:

$$f(t) = (\alpha\beta)t^{\beta-1} * \exp(-\alpha t^\beta), \quad \text{where } t > 0, \quad (13)$$

and a cumulative distribution function given by:

$$F(t) = 1 - \exp(-\alpha t^\beta), \quad (14)$$

where  $\alpha$  and  $\beta$  are parameters. Following Meyer, the Weibull distribution may be the most appropriate function for a failure law whenever an industry comprises a number of firms, and failure is essentially due to the most severely flawed firm among many flawed firms in the industry.

The reliability of the industry at time  $t$  is defined as  $R(t) = \Pr(T > t) = 1 - F(t)$ , which explains the probability that flour mills are still operating at time  $t$ . The hazard function (or failure rate) is defined as  $Z(t) = f(t)/R(t)$ , which simply represents a conditional probability or a transition probability that firms fail in period  $t$  given that they have survived through period  $t-1$ . The hazard function,  $Z(t)$ , is increasing in  $t$  for  $\beta > 1$ , constant for  $\beta = 1$ , and decreasing for  $\beta < 1$ . The hazard function corresponding to the exponential distribution is a special case of the Weibull distribution with  $\beta = 1$ . It should be noted that there is a mathematically equivalent specification in terms of a probability distribution for any specification in terms of a hazard function (11).

The estimated nonlinear cumulative Weibull distribution function is:

$$F(t) = 1 - \exp[-.020309380 * t^{1.044398130}], \quad R^2 = .99, \\ (.0034121389) \quad (.0766475454) \quad (15)$$

where numbers below coefficients are the estimated standard errors, and  $t = 0$  for the year 1973. Using this cumulative distribution function, we estimated reliability and hazard functions to be:

$$R(t) = \exp[-.020309380 * t^{1.044398130}], \quad \text{and} \\ Z(t) = .021211078 * t^{.044398130}. \quad (16)$$

Results indicate that about four mills will annually merge with existing mills, bringing down the total to about 160 mills by the year 2000 (table 6).



## Projecting the U.S. Wheat Flour Supply

To conduct simulation analysis using the estimated modified multinomial logit model, some assumptions are necessary for explanatory variables. Deflated aggregate disposable income grew 32 percent during 1973-87, and production worker hours declined by 25 percent during the same period. Therefore, our first scenario assumes that aggregate disposable income will increase by 2.5 percent annually and production worker hours will decline by 2 percent annually, while wheat price remains at the 1987 level (table 6).

Results indicate that the projected number of mills for the size classes  $S_1$ ,  $S_2$ , and  $S_3$  declines. However, the number of mills for the smallest size class,  $S_1$ , declines faster than for the other two classes. Even though the total number of mills declines over the period, the number of mills for the largest size class,  $S_4$ , is expected to increase. Given the trend of increasing per-capita wheat flour consumption, and an increasing population, expansion of the largest size mills would be necessary to meet increasing domestic demand for wheat flour.

Scenario 2 assumes that both aggregate disposable income and production worker hours mirror scenario 1, but wheat price is assumed to increase by 2 percent annually. Results indicate that the number of mills for the size classes  $S_1$ ,  $S_2$ , and  $S_3$  will decline, but at a slower rate than in scenario 1. The number of mills for the largest size category,  $S_4$ , will remain unchanged. As wheat flour price rises, consumer demand for wheat flour declines, offsetting the increase in per-capita wheat flour consumption and diminishing the incentive for structural adjustments.

Scenario 3 assumes that both aggregate disposable income and production worker hours match those in scenario 1, but wheat price is assumed to decrease by 2 percent annually. Our results show that the struc-

tural changes are similar to those for scenario 1, but occur at a faster rate. The number of mills for the size classes  $S_1$ ,  $S_2$ , and  $S_3$  would decline, while the number for the size class  $S_4$  would increase at a faster rate to meet increased consumer demand.

Since the daily active milling capacity of a flour mill in each size category remains stable (see table 2), daily wheat flour supply can be estimated by multiplying the number of mills by the daily average milling capacity of a flour mill in each size category. The average number of annual milling days (307 days) is then multiplied by the estimated total daily wheat flour supply to derive annual wheat flour supply. Under scenario 1 in table 7, wheat flour supply is expected to decline over the period with minor fluctuations, from 378.7 million cwt in 1990 to 375.7 million cwt in year 2000. Under scenario 2, wheat flour supply declines steadily from 382.8 million cwt in 1990 to 343.3 million cwt in year 2000, while it increases steadily under scenario 3 to 435.6 million cwt from 398.7 million cwt during the same period.

Wheat price can greatly affect the size distributions of the flour mill industry. As the government phases out producer subsidies, therefore, the structure of the U.S. milling industry may change significantly. Kim reported that the U.S. wheat price would not change, however, when major industrialized countries remove their producer subsidies (12). But, as shown in scenario 1, the expansion of the size distributions will continue due to automation in production and increased consumer demand. To determine if U.S. wheat flour supply under trade liberalization would be adequate, we assumed that a 1-percent annual growth rate in population during 1973-87 will continue to the year 2000. Under this assumption, population would climb to 277.4 million from 1987's 243.7 million. Therefore, per-capita wheat flour consumption in the year 2000 would be 136 pounds under scenario 1, 124 pounds under scenario 2, and 157 pounds under scenario 3.

Table 6—Estimated number of mills in each size category according to reliability theory

Year	Number of mills	Scenario 1 <sup>1</sup>				Scenario 2 <sup>2</sup>				Scenario 3 <sup>3</sup>			
		$S_1$	$S_2$	$S_3$	$S_4$	$S_1$	$S_2$	$S_3$	$S_4$	$S_1$	$S_2$	$S_3$	$S_4$
1990	203	48	60	44	51	47	60	44	52	44	59	44	56
1991	198	44	59	43	52	44	59	43	52	40	58	42	58
1992	194	40	59	41	54	42	59	41	52	36	57	40	61
1993	189	37	58	39	55	39	58	40	52	32	56	38	63
1994	185	34	57	38	56	36	58	38	53	28	55	37	65
1995	180	31	56	37	56	34	57	37	52	25	53	35	67
1996	176	29	55	35	57	32	56	35	53	21	52	33	70
1997	172	26	54	34	58	29	56	34	53	19	50	31	72
1998	168	24	53	32	59	28	55	33	52	16	49	29	74
1999	164	21	52	31	60	26	54	32	52	14	47	27	76
2000	160	19	51	29	61	24	53	31	52	12	45	25	78

<sup>1</sup>Aggregate disposable income increases by 2.5 percent annually, and production worker hours decrease by 2 percent annually, while wheat price remains at the 1987 level.

<sup>2</sup>Aggregate disposable income and production worker hours mirror scenario 1, and wheat price rises by 2 percent annually.

<sup>3</sup>Aggregate disposable income and production worker hours mirror scenario 1, and wheat price falls by 2 percent annually.

Table 7—Projected wheat flour supply under different scenarios

Year	Scenario 1 <sup>1</sup>	Scenario 2 <sup>2</sup>	Scenario 3 <sup>3</sup>
	<i>Million hundredweight</i>		
1990	378.7	382.8	398.7
1991	379.0	379.0	401.4
1992	382.5	374.3	408.5
1993	381.0	370.5	411.1
1994	381.7	369.8	415.9
1995	377.9	361.9	418.1
1996	376.7	360.8	425.1
1997	377.4	357.8	427.6
1998	376.2	350.2	430.6
1999	376.9	346.8	433.1
2000	375.7	343.3	435.6

<sup>1</sup>Aggregate disposable income increases by 2.5 percent annually, production worker hours decrease by 2 percent annually, and wheat price remains at the 1987 level.

<sup>2</sup>Aggregate disposable income and production worker hours mirror scenario 1, and wheat price rises by 2 percent annually.

<sup>3</sup>Aggregate disposable income and production worker hours mirror scenario 1, and wheat price falls by 2 percent annually.

With per-capita wheat flour consumption at 128 pounds in 1987, wheat flour supply under all scenarios would be adequate to meet increasing consumer demand.

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# Determination of a Variable Price Support Schedule as Applied to Agricultural Production Control

Wen-Yuan Huang and Bengt Hyberg

**Abstract.** *Adoption of variable price support (VPS) schedules could be effective in controlling agricultural production and targeting program benefits to specific farm groups. The design of a VPS program would require determination of price schedules for farm-level production decisions that satisfy both farmer and program objectives. We applied a primal-dual mathematical programming model to the determination of a VPS program for production control of U.S. corn, wheat, and soybeans. We show that government program costs under the VPS program would decline to \$15 billion from \$26.8 billion under a comparably scaled mandatory production control program. The program benefits to a 120-acre farm would increase 80 percent to \$18,000 from \$10,000, while the benefits to a 2,500-acre farm would fall 82 percent to \$40,000.*

**Keywords.** *Supply, agricultural policy, commodity programs, income support, primal-dual programming.*

U.S. agricultural commodity markets frequently experience excess supply or surplus, especially when prices are supported above the market price. Stock accumulations resulting from surpluses lead to increased government expenditures on farm support programs and a depressed farm economy. Continuous technological innovation in U.S. agricultural production, unstable export demand (11), and slow domestic demand growth aggravate the surplus problem.<sup>1</sup> To deal with this problem, the U.S. Government has often attempted to control agricultural supply by requiring commodity program participants to place cropland into set-aside programs or paid land diversion programs or by restricting farm sales of commodities. The success of these controls has been mixed (2).

## A Variable Price Support Program

A promising alternative to mandatory production control systems is the variable price support (VPS) program (4). Rather than restricting production on an individual farm basis, participating farms would face a set of declining support prices for the program crops. Under these support prices, a farmer would receive a monotonically declining price as his/her output of a particular crop increased. The price received for the initial units of production would not be affected by the total quantity produced. Figure 1 illustrates this con-

cept. The highest support price, ( $IP_1$ ), is paid to a farmer for the production of the first unit ( $w_1$ ) and the next highest price is paid for the next unit ( $w_2$ ), and the declining price continues. A VPS schedule that sets the support price for the last unit of the commodity produced ( $w_n$ ) equal to or below the expected market price would induce farmers to base marginal production (beyond  $w_n$ ) on the market price. Therefore, marginal production would be governed by market prices instead of the support prices. A farmer with an initial production level,  $q_{ij}^0$ , under current programs, responds to the market price by producing  $q_{ij}^1$  under the VSP program, a reduction of  $w_s$ . Because a VPS program allows market prices to prevail, chronic surpluses disappear.

The basic reasoning behind the use of a VPS can be compared with the logic of increasing block rate structure used by electric power companies and municipal water authorities. The utility rate structure is designed to discourage excessive electricity (water) consumption by consumers, and the VPS discourages excess production by farms. While the utility companies are concerned with finding a price schedule that leads to the efficient utilization of its physical capacity, the VPS is concerned with finding a price schedule that leads to a more efficient allocation of resources within the agricultural sector. Such an allocation would reduce the social welfare deadweight loss associated with excess production.

The objective of this article is to investigate methods of obtaining a set of VPS schedules to achieve predetermined national production levels. Two methods for estimating the farm-level price schedules will be presented. The first method employs an iterative procedure, while the second method uses a primal-dual (PD) programming model. The iterative method is presented because it illustrates the problem to be solved. The programming model is a generalized procedure. Both methods use a farm production decision model to estimate the production response at the farm level. The estimated commodity price schedules are declining functions of the quantity of the commodity produced on a farm.

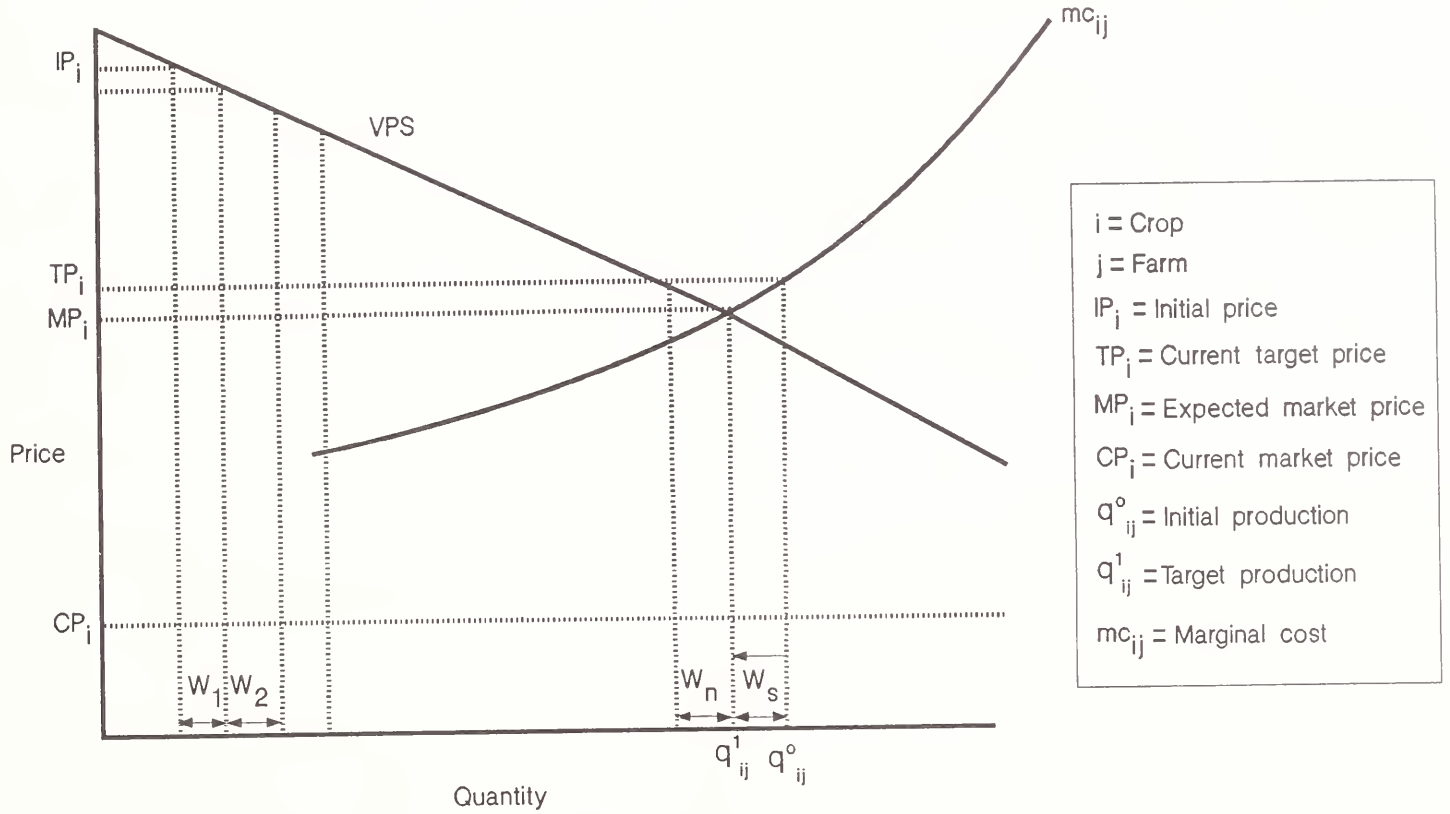
The PD formulation has the advantage of expressing the problem in a concise manner. For a simple farm decision model, the PD formulation can determine the price schedule in one iteration. If the PD formulation solves the problem, the farm decision model must be simple (linear in its constraints) to obtain a solution. The iterative procedure on the other hand is less elegant in its formulation but has the advantage of

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<sup>1</sup>Italicized numbers in parentheses cite sources listed in the References section at the end of the article.

Figure 1

# A linear declining price support schedule to reduce single-farm crop production



being able to estimate the price schedule when non-linear constraints are imposed. Thus, the iterative procedure can be used to determine the farm production response for a farm decision model that includes a risk component or has nonlinear constraints.

## The Problem

The problem can be formally stated as follows: given a desired aggregate production level,  $TQ_i$ , find a set of farm-level, declining support price schedules,  $P_i(a_i, q_{ij})$ , such that the sum of the production of crop  $i$  over all farms is equal to  $TQ_i$ . That is,  $\sum q_{ij} = TQ_i$  for all  $i$ . In this problem,  $q_{ij}$  is the commodity  $i$  produced by the farm  $j$ , and  $a_i$  is the parameter to be estimated. It is assumed that a farm produces a set of crops in a manner that maximizes net farm revenue.

The production decision model for farm  $j$  can be formulated as:

(Problem 1)

$$\text{Max } Z_j = \sum_{q_{ij}} \int_0^{q_{ij}} [P_i(\hat{a}_i, q) - \max(m_{pi}, c_{ij})] dq, \quad (1)$$

which is subject to the resource constraint:

$$\sum_i d_{ij} q_{ij} \leq L_j, \quad (2)$$

where  $P_i(\hat{a}_i, q)$  is the given government price support function for crop  $i$ ,  $q$  is the integration (dummy) variable for  $q_{ij}$  to be determined,  $m_{pi}$  is the expected market price for crop  $i$ ,  $c_{ij}$  is the production cost for crop  $i$  on farm  $j$ ,  $L_j$  is the land available for crop production, and  $d_{ij}$  is the portion of an acre on farm  $j$  required to produce one unit of crop  $i$ .  $\hat{a}_i$  is the parameter estimate that defines the support price function.

Given a set of estimated coefficients,  $\hat{a}_i$ , the optimal production response,  $q_{ij}^*$ , for farm  $j$  can be obtained. By solving problem 1 for all farms and summing  $q_{ij}^*$ , the aggregate production level,  $Q_i^*$ , can be determined. In practice, the problem for all the farms is solved simultaneously rather than repeatedly solving the problem for each farm. This is done by solving:

(Problem 2)

$$\text{Max } Z_2 = \sum_i \sum_j \int_0^{q_{ij}} [P_i(\hat{a}_i, q) - \max(m_{pi}, c_{ij})] dq, \quad (3)$$

which is subject to:

$$\sum_i d_{ij} q_{ij} \leq L_j \text{ for all } j. \quad (4)$$

The aggregate and farm-level production figures obtained from problem 2 will be identical to  $Q_i^*$  and  $q_{ij}^*$  obtained from problem 1. The Kuhn-Tucker neces-



sary conditions (9) for the optimal solution of problem 2 are given by the following relations:

$$P_i(\hat{a}_i, q_{ij}^*) - \max(mp_i, c_{ij}) - \mu_j^* d_{ij} \leq 0, \text{ for all } i \text{ and } j, \quad (5)$$

$$[P_i(\hat{a}_i, q_{ij}^*) - \max(mp_i, c_{ij}) - \mu_j^* d_{ij}] q_{ij}^* = 0, \\ \text{for } i \text{ and } j, \quad (6)$$

$$\sum_i d_{ij} q_{ij}^* - L_j \leq 0, \text{ for } j, \text{ and} \quad (7)$$

$$\left[ \sum_i d_{ij} q_{ij}^* - L_j \right] \mu_j^* = 0, \text{ for } j, \quad (8)$$

where  $q_{ij}^* \geq 0$  and  $\mu_j^* (\geq 0)$  is the shadow price of the resource  $L_j$ .

Given a production level,  $TQ_i$ , the set of parameters,  $\hat{a}_i$ , in the price function,  $P_i(\hat{a}_i, q)$ , should be estimated such that the sum of the commodity production over individual farms  $j$ ,  $Q_i^*$ , is equal to this targeted level. To solve this problem, an iterative procedure and a PD mathematical programming model can be employed.

### An Iterative Procedure

This procedure uses the farm production decision model and the iterative estimation method outlined in the flowchart (fig. 2). The first step is to select a func-

tional form for the VSP schedule that is able to direct government benefits to the farm groups the program is designed to help. Two characteristics of the functional form that affect the program's ability to direct government benefits to a given farm size group are its general shape and its slope. The general shape of the functional form determines how the government support is distributed among farm size groups. The current farm program has a single flat support price which provides greater support to large farms because all farm production receives the same support and the larger farms produce more. An exponentially decreasing schedule, on the other hand, would provide a high support price to the first units of production, but the support would decline rapidly as onfarm production increased. The exponential function directs a much greater proportion of benefits to smaller farms. A linear declining support price can direct a greater proportion of farm support payments to smaller farms than can current programs but not as much as under the exponentially declining rate.

Once the function,  $P_i(a_i, q)$ , has been selected, a preliminary VPS function,  $P_i(\hat{a}_i, q)$ , is specified using a set of starting values for  $\hat{a}_i$ . The production level,  $q_{ij}^*$ , for each farm is then determined using the farm decision model. By summing  $q_{ij}^*$  over all farms,  $Q_i^*$  is obtained. This level of production is then compared with the target production level  $TQ_i$ . If the difference for each commodity is not significant, the final set of  $\hat{a}_i$  has been found. Otherwise, the  $\hat{a}_i$  are adjusted and the set of adjusted  $\hat{a}_i$  is used to generate a new production estimate. A new set of  $\hat{a}_i$  can be computed on the basis of the inverse relationship between the value of  $\hat{a}_i$  and the quantity produced. The iterative process continues until  $Q_i^* \approx TQ_i$ , indicating a suitable set of  $\hat{a}_i$ .

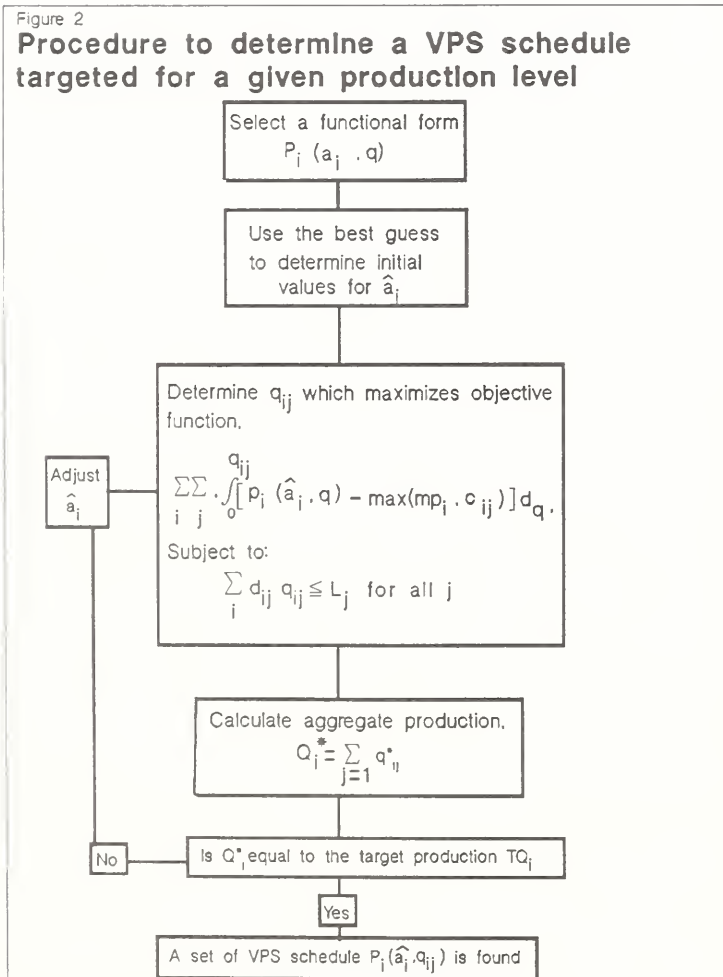
The procedure can be extended to estimate an efficient support price schedule given both a government farm program budget and target production levels. The procedure to find  $\hat{a}_i$ , however, becomes more complicated as additional restrictions are added. A PD programming model can be formulated as an alternative method to determine the VPS schedule.

### A Primal-Dual Programming Model

This approach uses the fact that the solution of a PD programming model is also the solution to the corresponding primal formulation. Thus, if the price schedule obtained by the PD model is used in the corresponding primal formulation, the production pattern and targeted aggregate production level,  $Q_i^*$ , obtained from the primal formulation will be identical to those produced by the PD model.

The PD problem is derived as follows (7):

1. A new objective function is constructed. The new objective function is the difference between the





objective functions of the primal and the dual formulations.

2. The constraints in the PD model include all constraints specified in the primal and dual formulations.

Problem 2 can also be expressed using a dual formulation (6, 12):

(Problem 3)

$$\begin{aligned} \text{Min } Z_3 = & Z_2 - \sum_i \sum_j [P_i(\hat{a}_i, q_{ij}) - \max(m p_i, c_{ij})] q_{ij} \\ & + \sum_j \mu_j L_j, \end{aligned} \quad (9)$$

subject to:

$$P_i(\hat{a}_i, q_{ij}) - \max(m p_i, c_{ij}) - \mu_j d_{ij} \leq 0, \text{ for all } i \text{ and } j, \quad (10)$$

where the term  $Z_2$  in equation 9 is the objective function of problem 2.

Given the primal and dual formulations and the above description of the PD formulation, the PD formulation for problem 2 is:

(Problem 4)

$$\begin{aligned} \text{Max } Z_4 = & \sum_i \sum_j [P_i(\hat{a}_i, q_{ij}) - \max(m p_i, c_{ij})] q_{ij} \\ & - \sum_j \mu_j L_j, \end{aligned} \quad (11)$$

subject to:

$$\sum_i d_{ij} q_{ij} \leq L_j, \text{ for all } j, \text{ and} \quad (12)$$

$$P_i(\hat{a}_i, q_{ij}) - \max(m p_i, c_{ij}) - \mu_j d_{ij} \leq 0, \text{ for all } i \text{ and } j. \quad (13)$$

A solution to problem 4, if it exists, will be a solution to problem 2 (7, 12).

Using problem 4 as the basic framework, the problem can be reformulated to find a set of proper values,  $\hat{a}_i$ , to determine a set of farm-level support price schedules. This is done by maximizing  $Z_5$  (problem 5) with respect to the parameters,  $a_i$ , as well as  $q_{ij}$  and  $\mu_j$ . It should be noted that in problem 5,  $a_i$  is a variable parameter.

(Problem 5)

$$\begin{aligned} \text{Max } Z_5 = & \sum_i \sum_j [P_i(a_i, q_{ij}) - \max(m p_i, c_{ij})] q_{ij} \\ & - \sum_j \mu_j L_j, \end{aligned} \quad (14)$$

subject to:

$$\sum_i d_{ij} q_{ij} \leq L_j, \text{ for all } j, \text{ and} \quad (15)$$

$$P_i(a_i, q_{ij}) - \max(m p_i, c_{ij}) - \mu_j d_{ij} \leq 0, \text{ for all } i \text{ and } j. \quad (16)$$

$P_i(a_i, q_{ij})$  is the price function that must satisfy the following two sets of conditions:

$$\partial P_i(a_i, q_{ij}) / \partial q_{ij} \leq 0, \text{ for all } i \text{ and } j. \quad (17)$$

and

$$P_i(a_i, 0) \leq b_i, \text{ for all } i. \quad (18)$$

where  $b_i$  is a constant for crop  $i$ . Condition 17 states that a support price cannot increase as production of a commodity on a farm increases. Equation 17 results in the declining support prices. The relationship in equation 18 sets the initial maximum support price for each commodity. A solution  $(a_i^*, q_{ij}^*)$  that satisfies problem 5 also satisfies problems 4 and 2. To incorporate supply control, aggregate production constraints are added to problem 5:

$$\sum_j q_{ij} = TQ_i, \text{ for all } i. \quad (19)$$

The addition of production constraints 19 to problem 5 changes the slope of the price schedule  $P_i(a_i^*, q_{ij}^*)$ .<sup>2</sup> However, the condition that the  $q_{ij}$  obtained from problem 5 is the solution to the individual farm's revenue maximization problem (problem 2) still holds. The condition holds because any set of values  $(a_i^*, q_{ij}^*)$  obtained from problem 5 with the production constraints 19 will also satisfy the Kuhn-Tucker conditions (relations 5 to 8) associated with problem 2. Thus, the price function obtained from this formulation can be substituted in problem 2 to obtain an identical production pattern.

The PD approach can lead to a nonlinear programming problem that becomes difficult to solve. For instance, use of a nonlinear price function or a budget constraint to control total program expenditures makes obtaining an optimal solution difficult. In some situations, the combination of a PD formulation with an iterative procedure is the only method to obtain the optimal VPS schedule to control production.

## A PD Model With Linear Price Schedules

We used a linear price function to design a declining support price schedule to control crop production. A PD model with the linear price function is formulated. We compared the results from the PD model with the results from a mandatory production control (MPC) program, assuming the market price to be less than the production costs for each crop.

We constructed a PD model with a set of production constraints and a linear price function. A linear,

<sup>2</sup>For example, with a linear functional form for the price support and a declining quantity of  $TQ_i$ , the slope becomes steeper.

declining price function is used because it has high initial support prices which are an advantage to small farms. The price function is expressed as:

$$P_i(a_i, q_{ij}) = b_i - a_i q_{ij}, \text{ for all } i \text{ and } j, \quad (20)$$

where  $b_i$  is a given positive constant and  $a_i$  is a positive parameter to be solved for.

Problem 5, containing a linear declining function 20, can be reformulated as:

(Problem 6)

$$\text{Max}_{a_i, q_{ij}, \mu_j} Z_6 = \sum_i \sum_j [-a_i q_{ij}^2 + (b_i - c_{ij}) q_{ij}] - \sum_j \mu_j L_j, \quad (21)$$

subject to:

$$\sum_i d_{ij} q_{ij} \leq L_j, \text{ for all } j \quad (22)$$

$$b_i - a_i q_i - c_{ij} - \mu_j d_{ij} \leq 0, \text{ for all } i \text{ and } j. \quad (23)$$

We add a set of constraints to control aggregate production:

$$\sum_j q_{ij} = Q_i, \text{ for all } i. \quad (24)$$

## An Application

The VPS program was compared with an MPC program similar to that proposed by Byrd and Harkin (1). An MPC program can be characterized as offering a flat support price for the controlled commodities while limiting the cropland acreage available for production on each farm. Each farm idles the same proportion of its cropland under an MPC program, and the quantity of a farmer's production does not affect the support price received for that commodity. For this reason, the shape of the rate schedule is horizontal or flat.

In this case study, a farm is considered a production unit of 100 acres or larger which can annually grow

corn, soybeans, wheat, or a combination of these three crops. U.S farms are divided into eight ( $j = 1, \dots, 8$ ) groups according to size with the farms in each group assumed to be identical. Costs of production for each commodity reflect economies of scale (table 1). We used the 1982 census data to estimate the number of farms and the average crop yield by farm size class.

To provide adequate income to each farm under the VPS program, we assume the initial prices ( $b_i$ ) for the 1986 crop year approached 80 percent of parity prices (the support price level proposed by Byrd and Harkin (1). Table 2 carries the initial prices. The equilibrium production associated with these prices came from the FAPSIM model (10). When production is reduced to the target level, the market prices, theoretically, equal the support prices. There would be no government payment to farms at these price levels. The government would have to pay farms participating in the VPS program only if the final market prices fall below the support prices. Under the MPC program, table 2's prices and production represent support prices and the quantities of production to be controlled. To achieve the targeted production level, each farm, regardless of its size, must idle the same proportion of land.

The Generalized Algebraic Modeling System (GAMS) (5), a nonlinear, quadratic programming package, estimated the price schedules and associated production response of individual farms under the VPS program. A linear programming model determined the production levels and farm incomes under the MPC program.

## Results

The estimated price functions for corn, soybeans, and wheat are  $P_1 = 3.95 - 0.000077q_{1j}$ ,  $P_2 = 9.76 - 0.00065q_{2j}$ , and  $P_3 = 5.36 - 0.000155q_{3j}$ , respectively. These price schedules produce higher net incomes for small farms with no increase in government expenditures relative to the MPC program. Under an MPC program, which has a single support price, a small

Table 1—Number of farms, acreage, crop yield, and production cost per farm size group

Farm size group (acres) <sup>1</sup>	Average acreage per farm, $L_j$	Number of farms, $N_j$	Yield, $Y_{ij}$			Production cost, $C_{ij}$ <sup>2</sup>		
			Corn	Soybeans	Wheat	Corn	Soybeans	Wheat
	<i>Acres</i>	<i>Thousands</i>	<i>-----Bu./acre-----</i>			<i>-----Dollars/bu.-----</i>		
100-139	120	67	101	28	34	1.87	3.99	3.19
140-179	160	69	102	30	33	1.84	3.91	3.12
180-219	200	50	102	30	35	1.80	3.82	3.06
220-259	240	48	104	31	34	1.76	3.73	2.98
260-499	280	161	107	27	34	1.72	3.64	2.91
500-999	750	97	110	31	34	1.68	3.57	2.86
1,000-1,999	1,500	57	111	30	35	1.65	3.50	2.80
2,000 and more	2,500	32	110	28	33	1.65	3.50	2.80

<sup>1</sup>It is assumed that only full-time farms can participate in the VPS. Excluded are farms with fewer than 100 acres because they are likely to be part-time farms with substantial off-farm income.

<sup>2</sup>The production costs are derived from a 1982 base solution of the National Linear Program LP model (3), adjusted for farm size from the study by Miller and Rodewald (8).



farm with 120 acres would receive \$10,500 in government support payments, while a large farm with 2,500 acres would receive more than \$220,000 in benefits. The VPS schedule directs more benefits to small farms. A small farm would receive \$18,500 in government benefits, while the maximum payment received by a large farm falls below \$40,000. Thus, net incomes for small farms increase substantially with a VPS program, while transfers to large farms decline.

The distribution of program benefits under an MPC program demonstrates the difficulty of flat support price schedules in supporting small-farm income. Because of their large production levels, large farms receive most of the government benefits. In addition, a set of flat support prices fixed above prevailing market prices will encourage profit-maximizing producers to increase production, creating excess commodity supplies and increasing program payments and government storage costs. The VPS program is designed to discourage excess production by removing program production incentives beyond some targeted production level,  $TQ_i$ . Production beyond  $TQ_i$  will be eligible to receive only a support price that is below the market price, resulting in marginal production decisions that are based on market prices.

If market prices are above the target price, program costs for both the MPC and VPS programs will be zero. Program costs will increase as the market prices slip from the target price. For example, if market prices (factored with the 1986 support loans) are \$1.98 for corn, \$4.88 for soybeans, and \$2.68 for wheat, an MPC program would cost \$26.8 billion, while a VPS program would cost the government \$15 billion.

Government expense under the VPS program is lower because the support price is monotonically reduced for each additional unit of production. With the marginal support price below the expected market price, production that exceeds the target quantity would not require storage. Government expense for the storage of commodity surpluses would decline. Administrative costs would remain constant because a VPS program could use existing program yields and program enrollment procedures. So, the VPS could diminish program costs by reducing the amount of production receiving support payments, the size of the marginal support payment, and storage costs.

## Conclusions

The VPS program would enable the government to control agricultural program spending while meeting commodity program objectives. The marginal support price at the target production level should be set below the expected market equilibrium price in designing a support price schedule. Government expenditures would be reduced whenever the marginal support price fell below the market price.

**Table 2—Target quantities and expected market prices for 1986 crop year under a mandatory supply control program**

Crop	Production, $TQ_i$	Prices, $b_i$
	<i>Million bushels</i>	<i>Dollars/bushel</i>
Corn	6,161	3.95
Soybeans	1,836	9.76
Wheat	2,136	5.36

The VPS program requires the estimation of price schedules that will lead to farm production decisions that satisfy both farmer and commodity program objectives. Both the iterative and PD mathematical programming procedures are useful tools for generating price schedules appropriate for a given VPS program. These modeling systems can design a VPS price schedule that achieves both national and farm-level commodity goals. The iterative procedure is a relatively simple procedure (compared with the PD method) which does not require an advanced modeling technique. The procedure, however, can limit determining an optimal support price schedule when production restrictions are added. The PD approach, on the other hand, can be used for the situation with multiple production restrictions, albeit requiring an advanced modeling technique to set up a PD problem that can lead to a difficult-to-solve nonlinear programming problem. In some situations, a combination of a PD formulation with an iterative procedure is the only way to obtain the optimal VPS schedule.

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# Nonfarm Prospects Under Agricultural Liberalization

Maureen Kilkenny

**Abstract.** *What does the United States stand to gain from liberalizing international trade in agriculture? This article estimates potential dollar gains and simulates the relocation of workers out of agriculture and into nonfarm activities. Different nonfarm sectors would expand under three cases of macroeconomic adjustment to the change in farm policies. The benefits of full liberalization would arise largely from the implied reduction in the Federal budget deficit. The greatest benefits would result if in addition to liberalization, macroeconomic policies that stimulate investment or net exports were pursued.*

**Keywords.** *Liberalization, nonfarm employment, computable general equilibrium modeling.*

Budget pressures, the new farm bill, and the Uruguay Round of the General Agreement on Tariffs and Trade (GATT) have fueled debate on how liberalizing agriculture will affect the United States. Does farm sector activity indeed raise the gross national product (GNP), particularly when subsidies add to the government budget deficit? If the United States liberalizes agriculture but reduces farm employment, could these farmworkers be employed more efficiently in other sectors?

This article presents estimates of the efficiency and employment implications for the nonfarm economy of complete multilateral agricultural liberalization under three plausible macroeconomic adjustment scenarios. Relative to 1986 (the latest year for which enough detailed data were available), multilateral agricultural liberalization may allow for real GNP gains of \$4.5 billion in the United States because about 200,000 full-time workers will move from farming to nonfarm sectors. Since the gains from freer trade and markets are achieved by a reallocation of economic resources, an economywide computable general equilibrium (CGE) model is appropriate for simulating reallocation and estimating possible gains. The simulation analyses indicate that benefits from multilateral liberalization have less to do with removing the policy distortions than with reducing the government deficit.

Agricultural liberalization is a catchall term meaning complete market orientation, which implies the termination of coupled farm income support, government stocking to support prices, restrictions on imports, all

export enhancement programs, and restrictions on participants' land use. Liberalization is multilateral when all other countries also end domestic support that affects trade, import protection, and export subsidies.

Previous multimarket and CGE analyses of liberalization found different comparative-static economywide gains for the United States. The WALRAS applied general equilibrium model (4) was used to estimate that real output in the United States could increase by 0.1-0.2 percent under unilateral liberalization (18).<sup>1</sup> This gain is \$7.4 billion relative to the 1986 real GNP of \$3.7 trillion. The Static World Policy Simulation (SWOPSIM) multimarket model estimated nominal GNP gains, relative to 1986, of about \$9 billion (21). Robinson, Kilkenny, and Adelman used a nonlinear Walrasian CGE model to estimate real GNP gains of about \$10 billion relative to 1991 for multilateral liberalization (19). Hertel, Thompson, and Tsigas used a log-linear CGE model to estimate that unilateral liberalization could result in a \$6.6-billion nominal GDP reduction but in a real domestic income gain of \$5 billion relative to 1982 (8).

Previous estimates may be biased, however, because of how factor supplies or government budgets are modeled. In most previous models, liberalization was simulated by assuming or imposing that farm program spending changes would not affect the government budget. For the WALRAS experiments, household income tax rates were allowed to fall in order to hold the government budget deficit constant. The SWOPSIM multimarket model does not model the government explicitly. Hertel, Thompson, and Tsigas assumed that saved revenues are redistributed back to households (8). Only Robinson, Kilkenny, and Adelman allowed both the savings and the tax revenue increases to reduce the U.S. Government budget deficit. I used the same approach for this article.

There are two possible paths of adjustment when the government budget deficit declines through saved farm program expenditures (13). In the first, more savings may be available for private domestic investment. Factors of production would be pulled into the investment goods-producing sectors. In the second, foreign capital investment may fall, which is consistent with the behavior of foreign investors who respond to a reduction in U.S. interest rates (and the value of the dollar) as government borrowing falls. Decreased foreign capital inflows would reflect a reduced trade defi-

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<sup>1</sup>Italicized numbers in parentheses cite sources listed in the References section at the end of this article.



cit. The reduced value of the dollar would dampen imports and stimulate exports. In the second case, factors of production would be pulled into the import-competing and export-oriented sectors.

The reallocation of labor across sectors can change GNP simply because labor productivity and wages differ across sectors. These differences are due, in part, to variations in labor-augmenting capital across sectors (10), and because some sectors are willing to pay higher real wages to elicit higher productivity (14, 15). These two factors are interrelated because workers in capital-intensive industries are generally paid higher wages (5, p. 78).

There are three ways to handle the observed variations in factor returns across sectors in CGE models. One approach, possible in all types of models, is to assume that these differentials are due entirely to immobility and to impose this assumption on the model. The other two ways allow for mobility but link the observed differences to the factor or to the sector. The former approach uses economywide data on compensation paid to the factor and quantity employed to calculate the economywide rate of return (22). Then the number of units of average return-earning factors employed by each sector is imputed as the compensation paid divided by the economywide average return, which associates the variations across sectors with the factors. The latter approach calculates sector-specific returns to each factor and then the proportional divergences paid by each sector from the economywide average return ( $WFDIST_{i,f}$ ; see "Nominal flow parameters," app. 1). The result associates the variations in factor returns with the sectors, which is consistent with the efficiency wage hypothesis and empirical evidence on the United States (16). This article relies on a version of the model in (19) which associates the variations in labor productivity with the sectors.

Another problem with previous CGE analyses concerns land supply. Total land available was assumed exogenous and fully employed. Liberalization entails removing set-aside constraints on land use. This was modeled by imposing that acreage once set aside would come back into production. This would necessarily increase real output. Furthermore, because land substitutes for labor, capital, or other inputs, estimates of outmigration of the other factors of production may be overstated.

Hertel, Thompson, and Tsigas modeled overall land supply inelasticity while allowing for elastic supplies of land for any one particular use (8). They detailed the farm sectors and specified a range of alternative uses of land, such as for crop production or pasture. Set-aside restrictions are modeled as *ad valorem* equivalents. This modeling specification may simulate the conversion of grain acreage, for example, into grazing land (8, p. 269). A simple version of this

assumption, which can be applied in models where the farm sectors are aggregated, is to distinguish cropland from total land supply, then simulate the levels of cropland use to maximize producer profits. Cropland use is thus endogenous but less than or equal to total available arable land supply, which is inelastic. I applied that approach.

CGE models are good tools for estimating the economywide impacts of sector-specific policies because farm policies, interindustry linkages, market distortions in the nonfarm economy, and the government and trade budgets are all explicit. I used the CGE model to replicate the pattern of production, employment, prices, income, and other variables in the U.S. economy in 1986 with the farm policies in place to provide a benchmark, then conducted experimental simulations. I calculate the influence of a policy as the difference between the simulation of the economy given the change and the benchmark solution.

Multilateral liberalization is simulated under three plausible macroscenarios. In the first scenario, called *transfer*, government expenditures on farm programs are redistributed as transfers to households (equal to providing decoupled income support or reducing household income taxes), so that there is no change in the government deficit or aggregate savings.

In the second, the *invest* scenario, the Federal Government's farm program expenditures are saved, reducing the deficit. More funds are available for domestic private investment (1). Since this is a shift between public and private domestic use of loanable funds, rather than a change in supplies or demands for funds, the rate of interest (exogenous) remains unchanged.

In the third scenario, the *balance of payments* (BOP), the demand for loanable funds declines because of the government budget deficit reductions. Although no asset markets exist in the CGE model, interest rates clearly could fall. The exchange value of the dollar would fall, foreign capital inflows would decline, and the current account balance would improve. The polar case of complete capital inflow offset is modeled by reducing foreign capital by the same amount as the reduction in the government deficit. The depreciation of the dollar is endogenously determined in the model, given this change in the balance of payments.

The discussion of these scenarios focuses on employment patterns and output in the nonfarm economy. This article reports which sectors displaced farmers would move to to achieve the optimal longrun pattern of resource allocation. The three patterns reported correspond to the three possible macroscenarios. The discussion provides a point of departure for the important task of quantifying adjustment costs. Farm sup-



port policies are “inextricably intertwined with the problem of factor market adjustment” (7, p. 4). Adjustment costs will depend on how much it costs to relocate, how much of farm skills are useful in nonfarm jobs, or how much retraining costs. Factor market adjustments to farm policy changes are also intertwined with monetary and fiscal policies, because these policies determine the sectoral composition of aggregate demand.

### The Model Assumptions

A version of the 10-sector USDA/ERS CGE model of the United States is used to conduct the experiments (20). (See (12) for an explanation of the farm program modeling.) The 10-sector model explicitly simulates only the five major U.S. farm policies: deficiency payments, loan and stocking programs, export enhancement, import quotas, and acreage restrictions. Since the 10-sector model generates the same overall real GNP level, pattern, and macroeconomic results as the 30-sector model (which disaggregates the food and fiber system more finely), it is just as useful and more clear as a model of the nonfarm economy. Appendix 1 shows the main model equations and parameter values.

The model distinguishes three agricultural sectors (dairy and meat, grains and oilseeds, and other agriculture), five industrial sectors (light consumer goods, basic intermediates, capital goods, construction, and electronics), and two service sectors (trade and finance, and other services) (app. 2).

The relevant differences among the sectors for these experiments are the character of demand facing the sector, the capital/labor ratio, and the factor returns in the sector relative to average returns (tables 1 and 2). These are the relevant differences because they indicate how policy changes influence employment. First, labor will move to sectors favored in the particular macroeconomic aggregate demand adjustment scenario. Second and all else equal, labor will move to more labor-intensive or high-productivity sectors. And, because productivity differs across sectors, these movements will spawn variations in estimated GNP across scenarios even though primary factor supplies change little.

The horizontal rows in table 1 show the character of demand. The dairy and meat sector has little to do with trade under the existing policies. Only 0.3 percent of that sector's output is exported, and imports constitute less than 1 percent of domestic sales. Final consumer demand equals 8.9 percent of the dairy and meat sector's output, which consists mainly of intermediate goods processed into light consumer goods. Grains and oilseeds are also intermediate goods (66 percent relative to output), but export demand accounts for 18.4 percent of the output from the farm-

**Table 1—Importance of trade, domestic consumption, investment demand, and intermediate demand, by sector**

Sector	E/XD	M/X	C/XD	I/XD	INT/XD
	<i>Percent</i>				
Dairy and meat	0.3	0.9	8.9	0	98.2
Grains and oilseeds	18.4	.2	2.0	0	66.2
Other agriculture	3.5	9.7	32.6	0	74.8
Light consumer goods	3.6	6.6	43.6	1.7	54.9
Basic intermediate goods	4.5	16.3	13.0	.8	96.6
Capital goods	13.0	17.6	15.8	32.1	42.9
Construction	0	—	0	61.5	19.7
Electronics	11.6	26.1	48.2	20.6	39.5
Trade and finance	2.4	—	53.9	4.1	37.5
Other services	5.8	4.5	43.6	.7	37.4

— = Does not apply.

E/XD is exports relative to production, M/X is imports relative to sales, and C/XD is private consumption relative to production.

XD is real domestic production.

I is investment demand relative to production, and INT is intermediate use relative to production. The percentage ratios will not sum to 100 because they have different denominators. Government consumption is not shown. Final demands C and I include imports.

Source: Model solution for 1986 benchmark.

gate. Other agriculture, which includes vegetables and fruits that are not processed, serves final consumer demand. Electronics, trade and finance, and other services are largely demanded by consumers, while the capital goods and construction sectors serve investment demand.

Exports are especially important in the grains, capital goods, and electronics sectors, where import competition is strong. Import competition also faces the other agriculture sector (sugar, tobacco, vegetables, fruit), where imports make up 9.7 percent of domestic sales.

Table 2 reveals the importance of the sectors in value added, real GNP, the relative factor returns in the sectors, and relative factor intensity. Given the size of the service sectors, the relative returns and intensity information can be interpreted in comparison with those sectors.

A ratio of profits or wages relative to the economywide average that is larger than 100 percent indicates that factors in a sector receive compensation for higher than average productivity or riskiness, or receive subsidies; and, factors are immobilized to some degree. Notice the relatively high return to capital in the light consumer goods sector (223 percent of average) provided by protection against imports, such as the Multifibre Arrangement concerning textiles and apparel, and quotas against imported processed dairy products and processed sugar. Those policies also keep the share of imports low relative to sales.

The variations in sectoral wage rates compared with the average reflect either that labor receives compensation in some sectors for higher productivity or the

Table 2—Sector contribution to value added, real GNP, relative factor returns, and relative factor intensity

Sector	Value added	Gross output	Rents	Wages	Capital/labor ratio
	--Percent--		---Ratio to average---		
Dairy and meat	0.30	1.20	121.2	58.7	2.5
Grains and oilseeds	.95	1.18	105.1	43.5	2.7
Other agriculture	.77	.78	78.0	56.3	.6
Agricultural results	2.02	3.16	106.1	53.7	
Light consumer goods	6.94	10.90	223.3	99.6	.4
Basic intermediate goods	9.94	13.84	155.8	138.4	1.4
Capital goods	5.07	8.37	50.7	145.4	.4
Construction	4.94	7.31	204.1	142.4	.2
Electronics	1.94	2.26	84.6	77.9	.2
Industrial results	28.83	42.68	154.1	126.8	
Trade and finance	16.85	14.47	128.6	96.2	.4
Other services	52.30	39.69	86.1	91.4	1.4
Services results	69.15	54.16	90.1	93.9	
Economywide results	100.00	100.00	100.0	100.0	1.0

Value added is value added by labor, land, and capital at product market prices. Gross output is real GNP originating in the sector in constant (1982) dollar terms. Rents are rates of return to productive capital by sector expressed as the percentage ratio to the economywide average. Wages are rates of return to labor by sector as a percentage ratio to the economywide average wage.

Source: Model solution for 1986 benchmark.

distortions in the labor market. Labor appears most productive in the basic intermediate goods, capital goods, and construction sectors.<sup>2</sup> Any changes that provoke labor to move out of agriculture will bring about an improvement in economywide productivity and an increase in GNP as typically measured. The source of the measured increase in GNP is known as a "composition effect" (3). The final column in table 2 indicates that the first two farm sectors are relatively capital-intensive. The relatively labor-intensive sectors are other agriculture, light consumer goods, capital goods, construction, electronics, and trade and finance.

For CGE models, these data represent general equilibrium. Wages and rents are endogenously determined to clear factor markets throughout the economy. The contributions of sector-specific productivities, factor market rigidities, and risk to relative factor returns, however, are held exogenous across sectors in all experiments.<sup>3</sup> The assumed Cobb-

<sup>2</sup>Factor productivity differences are typically measured from data on constant dollar value-added shares to labor, employing full-time equivalent units of labor (11). The same data are used in the CGE model to measure relative factor return differences.

<sup>3</sup>For contrasting views of the interactions between farm policies and risk, see (6, 8).

Douglas production technology rules out any factor-intensity reversals.<sup>4</sup> All else equal, labor released from agriculture will move to sectors where it is paid for being most productive or used relatively intensively.

## Real GNP Under Multilateral Liberalization

The basic liberalization experiment consists of dismantling import protection policies, constraints on land use, and terminating the \$26 billion spent on farm income and price supports. Liberalization in other countries is modeled by using estimates of world price and trade changes from the SWOPSIM global agricultural sector model (21) to update import demands and export supplies to the rest of the world, variables that are exogenous to this model.

Liberalization is simulated assuming sectorally mobile labor and immobile capital and endogenous land supply. When the restrictions on land use end, a level of cropland demand is chosen to maximize profit from production, while a level of cropland supplied is chosen to maintain the rate of return to cropland. This procedure assumes a perfectly elastic longrun supply of cropland, which may be converted to pasture. Given this mix of assumptions about factor mobility, the solutions represent medium-run to longrun equilibria.

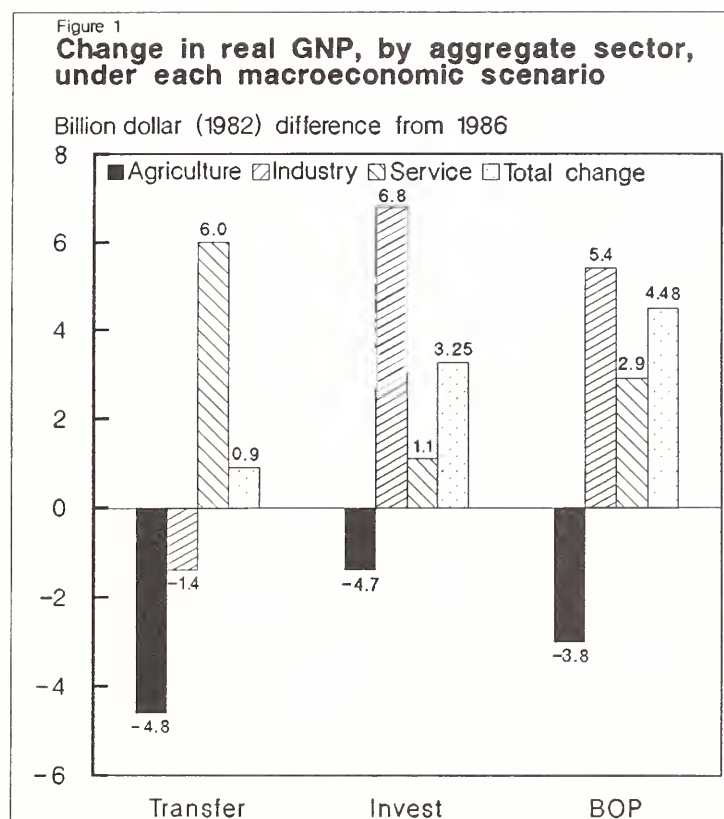
The SWOPSIM model (and all global models) estimates that multilateral liberalization enhances demand for U.S. grains and oilseeds, causing an increase in world prices which is passed on to the U.S. market price. The domestic price increase mitigates, but does not fully offset, the effects of lost income and price supports on net revenue from production. The overall impact is a downsizing of gross returns to labor, capital, and land in agriculture.<sup>5</sup> The main farm sector adjustments are that land supplied for crop production diminishes and some labor moves out of the farm sector.

The magnitude of economywide efficiency gains from liberalization depends critically on the macroeconomic adjustment to the change in farm program expenditures. Figure 1 and table 3 present the simulated changes in economywide real GNP, by aggregate sectors and overall, for each of the three macroscenarios. The fourth bar in each set of figure 1 indicates the change in real (constant 1982 dollar) GNP for each scenario.

<sup>4</sup>The simulation results are robust with respect to various homogeneous technology specifications. This is because the substitutability between primary factors of production is moot as long as relative factor prices do not change. Since agriculture is a small employer of labor, and cropland supply is modeled as elastic at prevailing land prices in the long run, even a drastic reduction in the demand for primary factors in agriculture will not affect longrun relative factor prices or cost shares.

<sup>5</sup>For discussions on the farm sector price, quantity, and income effects of unilateral or multilateral liberalization, see (2, 8, 21).





Under the *transfer* scenario, agriculture and industry contract, services expand, and overall GNP climbs slightly. If unspent farm program money is not saved, there are insignificant economywide gains of \$90 million real GNP (relative to a \$4,000-billion economy). The farm program money, when saved, can be either invested domestically (*invest*) or allowed to offset foreign capital investment (BOP). In both the *invest* and BOP scenarios, real GNP increases \$3.3 billion and \$4.5 billion (1982 dollars), respectively, demonstrating the importance of reducing the government deficit. Multilateral liberalization of agriculture alone is not the most important policy prescription for real economic growth.

Why is it that real GNP barely improves when farm program savings are withheld from reducing the deficit? There are two interrelated reasons. First, the income transfer part of the policy change simply substitutes one class of consumer demand (from farm households) for another (from taxpayers, which includes farm households). Thus, the composition of aggregate demand remains the same under the *transfer* scenario. Final consumer demand is largely for services (note the pattern in CLES<sub>i</sub> in app. 1). Second, since productivity in the service sector is generally below average but higher than in agriculture (table 2), overall GNP improves only slightly as labor moves from agriculture to services.

Efficiency gains are much higher under the *invest* and BOP scenarios because of the type of final demand

Table 3—Real GNP changes under multilateral liberalization, by macroscenario

Sector	Scenario <sup>1</sup>			
	Base	Transfer	Invest	BOP
<i>Billion dollars</i>				
Dairy and meat	12.2	11.87 (-2.70)	11.84 (-2.92)	11.91 (-2.38)
Grains and oilseeds	43.1	39.88 (-7.48)	39.84 (-7.56)	40.29 (-6.52)
Other agriculture	34.0	32.99 (-2.96)	32.93 (-3.14)	33.29 (-2.09)
Agricultural results	89.3	84.7	84.6	85.5
Light consumer goods	240.0	238.37 (-1.63)	237.87 (-2.13)	238.88 (-1.12)
Basic intermediate goods	346.8	346.65 (-.15)	347.88 (.13)	349.21 (.24)
Capital goods	209.7	209.88 (.18)	212.80 (.31)	212.76 (.06)
Construction	202.4	202.43 (.03)	206.59 (.17)	202.31 (-.09)
Electronics	74.6	74.78 (.22)	75.15 (.45)	75.74 (.10)
Industrial results	1,073.5	1,072.1	1,080.3	1,078.9
Trade and finance	658.8	660.29 (.23)	659.57 (.12)	659.19 (.06)
Services	1,890.2	1,894.75 (.43)	1,890.58 (.02)	1,892.70 (.13)
Services results	2,549.0	2,555.0	2,550.1	2,551.9
Total real GNP	3,711.8	3,711.89	3,715.05	3,716.28
Change from base: <sup>2</sup>		0.09	3.25	4.48

<sup>1</sup>Percent change relative to benchmark in parentheses.

<sup>2</sup>Real GNP in constant (1982) billion dollars.

stimulated. The construction and capital goods sectors expand. Labor generates relatively high real output in the sectors that supply investment goods. This shift of labor to higher productivity uses has a positive composition effect on real GNP.

The most balanced pattern of nonfarm expansion is under the BOP scenario. The real depreciation associated with the change in the balance of payments stimulates U.S. exports, which are produced by a variety of sectors including grains and oilseeds, capital goods, and electronics. To meet the demand, more land is kept in grain and oilseed production, and labor moves to the capital goods sector. Labor productivity appears quite high in the capital goods sector. These two patterns of resource use account for the relatively large real GNP gains.

The BOP scenario provides the highest overall economywide gains of about \$4.5 billion by terminating about \$26 billion in farm program expenditures. The implication is that for every \$100 of deficit-reduction savings due to ending farm programs and reducing the trade deficit, the economy gains \$17.20 of additional real GNP.



## Employment Patterns Under Multilateral Liberalization

The pattern of employment under liberalization is very sensitive to the macroscenario (fig. 2). Sectoral employment changes differ across the three macroscenarios. In each case, labor moves out of the agricultural and light consumer goods (food and feed processing) sectors. The longrun assumption of full employment means that all displaced farmers are re-employed elsewhere in the economy, a relocation that results in real GNP gains.

Why and where do factors relocate? Liberalization pushes labor out of agriculture because farm value added falls. Most of the program crops are in the grain and oilseed sector, and these sectors contract when subsidies are eliminated. The reduced supply raises farm-level market prices well above market prices under the programs. The higher grain and oilseed market prices ease the strain on crop sectors but hurt livestock sectors. High feed crop prices mean increases in intermediate costs in the dairy and meat sector. Since the rise in intermediate costs exceeds the increase in market price, value added declines. This

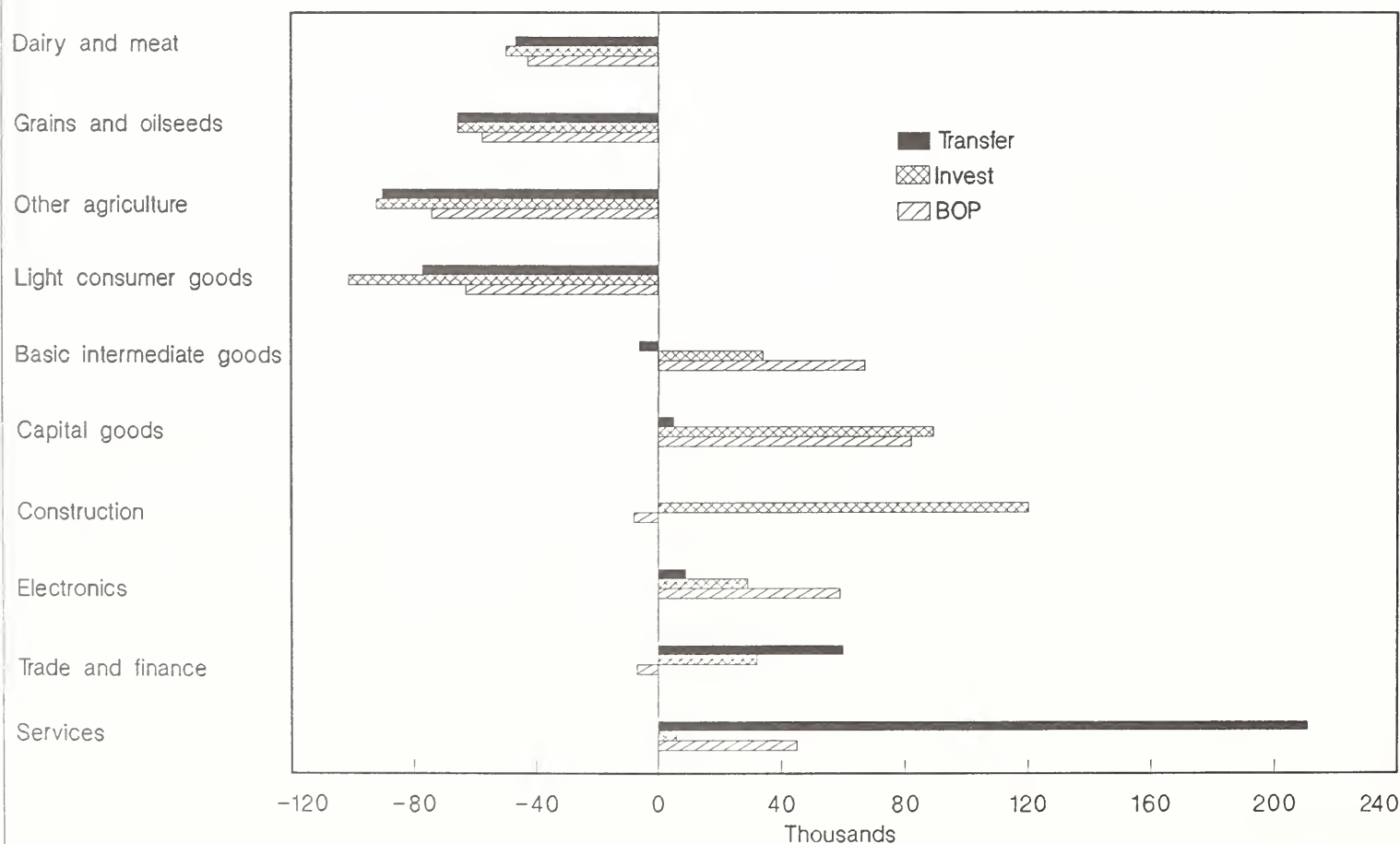
reduces the sector's return to labor, so labor moves elsewhere and dairy and meat sector output contracts.

Liberalization's effect on employment among agricultural sectors, however, depends only slightly on the macroscenario. As noted above, the grain and oilseed sectors retain more resources under the BOP assumptions because export demand is strong. Foreign demand strengthens farm market prices so much that value added per unit of grain output after multilateral liberalization is only 4 percent lower than in the base year, according to our model. That suggests that domestic farm market prices rise almost enough under multilateral liberalization to offset the loss of large deficiency payments and price supports.

Farm price increases are costly to some nonfarm sectors. Increases in market prices for agricultural products are passed on to the food processing industries in the light consumer goods sector as higher costs for intermediate goods, squeezing value added in the light consumer goods sector. Multilateral liberalization pushes 101,000 full-time equivalent workers (under the *invest* scenario) out of the light consumer goods sector (fig. 2).

Figure 2

**Relocated full-time equivalent workers across sectors under multilateral liberalization under three macroeconomic scenarios**



Labor moves to capture relative wage gains in the growing sectors, which are determined by the macroeconomic policies concomitant with liberalization. Under the *transfer* scenario, most displaced labor moves to services. Construction and capital goods industries are favored under the *invest* scenario because these sectors supply investment goods. These labor-intensive industries provide a strong pull to re-employ agricultural labor, so that there are almost 20 percent more job changes under the *invest* scenario than under the BOP assumptions.

The basic intermediate goods and electronics manufacturing sectors are favored under the BOP scenario. These are important import-competing sectors, and the depreciation improves their competitiveness by making imports appear more expensive in dollar terms.<sup>6</sup> These sectors also display high labor productivity. The expansion of these high-productivity sectors explains the larger real GNP gains under the BOP scenario. The gain equals about \$17,700 in additional real output per worker who changes jobs, the most optimistic estimate among the three macroscenarios.

An important determinant of adjustment costs is the number of job changes required to reach the new equilibrium. The high number of job changes (309,000) economywide required for an optimal allocation under the *invest* scenario may mean higher adjustment costs than for the BOP or transfer scenarios. The lowest number of job changes (253,000) economywide is required for the BOP scenario. The model predicts that 175,000-208,000 workers are expected to leave agriculture simply due to changes in policy during the 5- to 10-year liberalization period. The simulations do not include estimates of the changes in agricultural employment from causes other than agricultural liberalization such as technical change.<sup>7</sup> For example, forces not modeled here induced 279,000 workers to leave agriculture between 1980 and 1985, even with farm programs (23).

## Conclusions

Agricultural liberalization would likely benefit the economy, and certain nonfarm sectors may expand under multilateral termination of farm support programs. The analysis suggests that agricultural support programs have been a drag rather than a stimulus on GNP, especially given the Federal budget and trade deficits. Terminating programs multilaterally may allow the United States to benefit from real GNP gains of \$4.5 billion (1982 dollars).

In 1986, the United States spent \$31.4 billion on farm programs. Liberalization concomitant with government deficit reduction would not only save that amount but would also generate \$3.25-\$4.48 billion of additional real GNP. If we ignore the crowding out due to deficit spending, liberalization would generate \$0.09 billion of real GNP. This suggests that deficit spending is much more costly in terms of GNP forgone than farm programs.

This analysis shows that the government budget deficit did more to retard general economic growth than the 1985 farm bill, even in the peak farm program spending year of 1986. Deficit spending stimulates GNP when there is unemployment, but stimulates currency appreciation and trade deficits when the economy is fully employed (as in the recent past). The corollary: multilateral liberalization alone provides small overall benefits. If farm policy is liberalized, but the farm program savings are simply redistributed to households, the gains are insignificant. Only when the savings reduce the government budget deficit do significant gains occur.

The analysis relative to 1986 must be re-interpreted for the 1990's. The government budget deficit continues high, but unemployment has also been increasing. Thus, the effects of liberalization in the 1990's would probably be lower than the effects estimated relative to 1986. Real GNP gains due to liberalization may approach \$1.25-\$1.72 billion (constant 1982 dollars).

In the long run, agricultural liberalization will likely cause relocation of up to 208,000 workers from the farm sector and up to 101,000 from the food and fiber sectors to industry and/or service sectors. If liberalization occurs with real currency depreciation, the trade sectors may offer more employment opportunities. Barring depreciation, the reduced Federal budget deficit should spur private investment demand, stimulating employment in construction and other capital goods sectors.

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<sup>6</sup>This analysis ignores potential pricing-to-market noncompetitive strategies that may be used by foreign exporters to maintain their U.S. market share. If foreign suppliers price to market, they adjust their own prices to maintain constant dollar prices.

<sup>7</sup>Hertel and Tsigas note that labor outmigration from agriculture due to technical change averages 4.3 percent of the agricultural labor force (about 72,000) per year, or 720,000 in 10 years.



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## Appendix 1—CGE model

Indices:  $i, j$  = sector,  $f$  = factor,  $hh$  = household,  $ins$  = institution

### Market quantity

$FE_{i,f}$	Factor employment	$WF_f$	Market price
$X_i$	Composite good supply	$P_i$	
$XD_i$	Domestic output	$PX_i$	
$XXD_i$	Domestic sales	$PD_i$	
$E_i$	Exports	$PE_i$ domestic; $PWE_i$ world	
$M_i$	Imports	$PM_i$ domestic; $PWM_i$ world	

### Demand

$CD_i$	Final demand for private consumption
$DK_i$	Final demand for fixed investment
$DST_i$	Inventory investment by sector
$GD_i$	Final demand for government consumption
$GDTOT$	Aggregate real government consumption
$ID_i$	Final demand for investment goods
$INT_i$	Intermediate input demand

### Income/expenditures

$PVA_i$	Value added per unit output	$INDTAX$	Indirect tax revenue
$DEPRECIA$	Total depreciation charges	$INVEST$	Total investment
$ENTSAV$	Enterprise savings	$NETSUB$	Total export subsidies
$ENTTAX$	Enterprise tax revenue	$REMIT$	Net remittances
$FBOR$	Net foreign borrowing	$SAVINGS$	Total savings
$FXDINV$	Fixed capital investment	$SSTAX$	Social security revenue
$FSAV$	Foreign savings	$TARIFF$	Tariff revenue
$GENT$	Transfers to enterprises	$TOTHHTAX$	Household tax revenue
$GOVSAV$	Government savings	$YF_f$	Factor income
$GR$	Total government revenue	$YH_{hh}$	Household income
$HHSAV$	Total household savings	$YINST_{ins}$	Institutional income
$HHT$	Transfers to households		

### Policy instruments

$DEFPAY_i$	Deficiency payments
$ITAX_i$	Indirect business tax rate
$PIE_i$	Producer incentive equivalent
$PL_i$	Loan rate
$QR_i$	Import quota
$TE_i$	<i>Ad valorem</i> export subsidy rate
$TM_i$	<i>Ad valorem</i> tariff rate
$TMQ_i$	Quota premium (domestically received)
$TP_i$	Target price
$XP_i$	Program participating output
$XS_i$	Commodity stocks

### Economywide variables

$EXR$	Exchange rate	$FS_f$	Factor supplies
$PINDEX$	Price index	$RGNP$	Real gross national product

## Parameters

### Supply

$\alpha_{i,f}$	Factor share in value added
$AD_i$	Production function shift
$AT_i$	Supply shift
$\gamma_i$	Export share parameter in CET supply
$IO_{i,j}$	Input-output coefficients
$\eta_i$	Export-domestic transformation in CET exponent

### Demand

$AC_i$	Demand shift
$CLES_{i,hh}$	Expenditure share
$GLES_i$	Government expenditure share
$\delta_i$	Import share parameter in CES demand

<i>Demand</i>	
$\sigma_i$	Import-domestic substitution elasticity
$\rho_i$	CES exponent = $1/1+\sigma$
$E00_i$	Foreign demand shift
$\epsilon_i$	Foreign demand elasticity
$\partial_i$	CCC stocking parameter
$IMAT_{i,j}$	Capital composition coefficients

<i>Nominal flows</i>	
$WFDIST_{i,f}$	Proportion of sector to average
$SINTYH_{hh,ins}$	Household share of net income
$MPS_{hh}$	Household savings rate
$HTAX_{hh}$	Household income tax rate

*Selected parameter values*

Sector	$\eta$	$\gamma$	$\sigma$	$\delta$	$\partial$	$\epsilon$	$CLES_i$	$GLES_i$
Dairy and meat	3.0	0.01	4.0	0.26	3.0	3.0	0.33	0.06
Grains and oilseeds	1.3	.43	4.0	.18	3.0	5.0	.04	1.04
Other agriculture	1.5	.16	4.0	.40	3.0	—	.66	.18
Light consumer goods	1.5	.17	2.0	.21	—	—	14.60	3.26
Basic intermediate goods	1.5	.21	.75	.07	—	—	5.56	3.68
Capital goods	1.5	.30	.75	.07	—	—	3.61	10.66
Construction	1.7	*	—	—	—	—	—	12.35
Electronics	1.5	.28	1.1	.25	—	—	3.20	2.02
Trade and finance	2.7	*	—	—	—	—	21.70	2.72
Other services	2.7	.01	.5	*	—	—	50.30	64.03

\* = very close to zero. — = not applicable.  $CLES_i$  and  $GLES_i$  each sum to 100 percent.

**Equations**

*Domestic farm programs*

$DEFPAY = (TP-PL) \cdot XP$	Each program sector
$XS = XS00 + XS0 \cdot (PX/PL)^{-\partial}$	Each stocked sector
$PIE = [DEFPAY + (PL-PX) \cdot XS]/XD$	Each program sector
$PVA_i = PX_i \cdot (1-ITAX_i) - \sum_j IO_{j,i} \cdot P_j + PIE_i$	Each program sector

*Production, factor demand, and supply*

$XD_i = AD_i \cdot \Pi_f \cdot FE_{i,f}^{\alpha_{i,f}}$	
$WF_f \cdot WFDIST_{i,f} \cdot FE_{i,f} = \alpha_{i,f} \cdot PVA_i \cdot XD_i$	
$INT_i = \sum_j IO_{i,j} \cdot XD_j$	
$XD = AT \cdot [\gamma \cdot E^\eta + (1-\gamma) \cdot XXD^\eta]^{1/\eta}$	All sectors

*Demand*

$ID_i = \sum_j IMAT_{i,j} \cdot DK_j$	
$CD_i = [\sum_{hh} CLES_{i,hh} \cdot (1-MPS_{hh}) \cdot YH_{hh} \cdot (1-HTAX_{hh})]/P_i$	
$GD_i = GLES_i \cdot GDTOT$	
$X = AC \cdot [\delta \cdot M^{-\rho} + (1-\delta) \cdot XXD^{-\rho}]^{-1/\rho}$	All goods markets

*Trade and trade policies*

$M/XXD = [PD/PM \cdot \delta/(1-\delta)]^{1/(1+\rho)}$	Each import good
$E/XXD = [PE/PD \cdot (1-\gamma)/\gamma]^{1/(\eta-1)}$	Each export good
$E = E00 \cdot [(PE/EXR \cdot (1+TE))/PWE]^{-\epsilon}$	Large export sectors
$PM = EXR \cdot PWM \cdot (1+TM+TMQ)$	All import sectors



### Nominal flows

$$YF_f = \sum_i WF_f \cdot WFDIST_{i,f} \cdot FE_{i,f}$$

$$YINST_{labor} = YF_{labor} - SSTAX$$

$$YINST_{proprietors} = YF_{land} - FRETAX$$

$$YINST_{enterprise} = YF_{capital} + GENT - ENTSAV - ENT TAX - DEPRECIA$$

$$YH = \sum_{ins} (SINTYH_{hh,ins} \cdot YINST_{ins})$$

$$GR = TARIFF + FRETAX + IND TAX + TOTHTAX + SSTAX + ENT TAX + FBOR \cdot EXR$$

$$GOV SAV = GR - [\sum_i P \cdot GD_i + GENT + HHT + \sum_i PIE_i \cdot XD_i]$$

$$SAVINGS = HHS AV + GOV SAV + DEPRECIA + FSAV \cdot EXR + ENT SAV$$

### Market clearing

$$FS_f = \sum_i FE_{i,f}$$

f=labor and capital

$$P \cdot X = PD \cdot XXD + PM \cdot M$$

All goods markets

$$PX \cdot XD = PD \cdot XXD + PE \cdot E$$

All goods markets

$$X = INT + CD + GD + ID + DST$$

All goods markets

$$\sum_i PWM_i \cdot M_i = \sum_i PWE_i \cdot E_i + FSAV + REMIT + FBOR$$

$$SAVINGS = INVEST$$

### GNP and price index

$$GNP = \sum_i (PVA_i - PIE_i) \cdot XD_i + IND TAX + TARIFF$$

$$RGNP = \sum_i CD_i + DST_i + ID_i + GD_i + E_i - M_i$$

$$PINDEX = GNPVA/RGNP$$

## Appendix 2—Sector aggregation

<i>Sector</i>	<i>Major industries</i>	<i>BEA industry classification</i>
Dairy and meat	Milk, eggs, meat animals, poultry	1-1.03
Grains and oilseeds	Wheat, corn, rice, soy, cotton, peanuts, flax	2.01, 2.0201-2.0203, 2.06
Other agriculture	Sugar, tobacco, fruits, vegetables, nuts, other	2.03, 2.04-2.0503, 2.07, 3.0, 4.0
Light consumer goods	Food and kindred products, leather, footwear, feed, textiles, apparel, furniture, containers, printing	14-26, 33-34
Basic intermediate goods	Mining, petroleum, chemicals, plastic, rubber, glass & stone, iron & steel, fabricated metals	5-10, 27-32, 35-42
Capital goods	Munitions, engines, machinery, communications, trucks, motor vehicles, some electrical	13, 43-50, 52-54, 56.03 56.04, 57.03, 59-61
Construction	Private & government construction	11-12
Electronics	Office equipment, household appliances, semi-conductors, equipment, miscellaneous electronics, TV, radio, other	51, 55, 56.01-56.02, 57-57.02, 58, 62-64, 81, 84-85
Trade	Wholesale and retail trade, banking and insurance	69-70
Services	Real estate, services, noncomparable imports, transportation, and government	65-68, 71-79, 80, 82

Note: BEA industry classification from Appendix B: "Industry Classification of the 1977 Input-Output Tables," *Survey of Current Business*, p. 80, May 1984.

*A Taste of the Country: A Collection of Calvin Beale's Writings.* Edited by Peter A. Morrison. University Park: The Pennsylvania State University Press, 1990, 260 pages, \$28.50.

*Reviewed by Sonya Salamon*

Let's face it. Demographers write useful papers, but the prose involved in reporting numbers is typically not very engaging. A singular exception has been Calvin L. Beale who, during a career spanning almost 40 years as a demographer with the U.S. Department of Agriculture, has, with style and grace, produced oft-cited papers and books that track rural peoples and places. It is no accident that newspaper reporters, in particular, seek Beale out. He always rewards them with a pithy and quotable aphorism that succinctly captures the human story of changes within the populations, the small towns, and the economy of rural America. Seldom can a highly respected professional in a field also speak to the general public in words easy to understand. Beale, however, reads the census like a novel and translates the plot into well-turned phrases that are highly informative and jargon-free. His ability to do this derives from his uncanny encyclopedic knowledge of rural geography, history, and demography and a gift for synthesizing these data, enabling him to place new numerical facts in the appropriate spatial, economic, and social context.

Due to his position as a government demographer, Beale's work has more often appeared as congressional testimony, talks, or Economic Research Service reports than in the more easily accessible form of academic journal articles.

Thus, Morrison, the editor, performs a commendable service by gathering together gems by Beale that, while published, would require real detective work to uncover. Such papers might have remained fugitive documents had Morrison not included them in this volume. Of particular interest is a sprinkling of Beale's field notes, written in conjunction with his "busman's holidays" throughout the Nation's countryside. Beale devotes these travels, during which he has visited half of the 2,400 nonmetropolitan counties in the country, to poking about in cafes, cemeteries, churches, and towns. He interviews people or makes observations about the remnants of the past, such as abandoned buildings or the transition from German to English on tombstones, or such changes as community newcomers that tell the human story behind the census record of population fluctuations, persistence, or anomalies. Because of the relative obscurity and remoteness of

rural populations in this vast country, emerging patterns and trends tend to escape the notice of all but the most astute observer. The field notes, while somewhat amusing, also reveal the deep respect Beale has for the uniqueness of local peoples. The book lacks, unfortunately, any examples of Beale's trademark courthouse photographs taken in each county seat he visits.

The book is divided into three parts and gathers a wide range of demographic issues. Two perspectives emerge in Beale's work. Policy issues tend to thread through the fabric that charts overviews of rural diversity, whether regional or topical. Beale keeps in mind the policymakers inside the Washington, DC Beltway as he describes important differences in issues, such as the growth of rural populations or poverty, differences that might escape the notice of those who do not travel the countryside and talk to people. The other focus is on a sequence of topics that has intrigued Beale by being odd or counterintuitive. Such issues have occupied him in his travels over the years and connected him with researchers in other disciplines. It was, for example, an interest in the variation of rural midwesterners' fertility that caused our paths to cross in the late 1970's, when Beale attended a presentation of mine on ethnic differences in Illinois farm-family land transfers.

Included in the book's first section are two of the regional issues that motivated Beale's investigative activities. Beale had noted and mapped a correspondence of high fertility, ethnicity, and religious affiliation among midwestern farmers. Verifying the origins of pockets of ethnic farmers required much digging. After immigration from Europe declined around the turn of the century, census reports, until 1980, did not provide accurate information about ethnicity. Beale observed that, after 1970, high rural fertility had shifted out of the South to the North-Central region, and he sought an explanation. Using county atlases of township plat maps, Beale demonstrated the close connection between the persistence of Catholic ethnic groups among farmers, for example, and high fertility in local enclaves. Most of the data are published here for the first time. This paper illuminates Beale's unique ability to combine historical and local sources to explain patterns no one else would have thought to consider important. Also included in the section is a little known paper, reprinted from a 1972 *American Anthropologist* article, that deals with obscure mixed-racial populations found primarily in the U.S. South who, in the terminology of the period, Beale described as "white-Indian-Negro" (p. 33). Beale locates a number of the groups, explains their origins, lists their many names, and comments on the chances of their

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persistence. It is this paper that contains a prime example of the wit in Beale's work. After citing in his introduction the highly negative observations about the character of the Melungeon people, made by a woman who had lived among them, as "thieving," "untruthful," and "exceedingly immoral," Beale pens the comment: "Miss Drumgoole was essentially a sympathetic observer."

The second section is devoted to Beale's work in characterizing the economic and demographic transformations of the 20th century. These selections are particularly illustrative of how Beale used his geographic knowledge to combat what he terms "Procrustean policy beds, of the one-size-fits-all variety . . . all too common in public affairs" (p. 59). The pieces gleaned from various book chapters, congressional testimony, and reports focus on the scheme Beale developed for characterizing the country by 26 subcultural regions, distinguished by variation in their settlement patterns, economic activities, environmental resources and constraints, and ethnic makeup. A second chapter is concerned with a continuing Beale interest—rural communities in the North-Central region. With the Federal Government's rural policy in mind, Beale shows that misconceptions abound about all rural communities "dying," and the effect of decline or stability where each occurs. Another selection of the amusing and interesting results of field trips closes out the section, illustrating Beale's essentially anthropological interest in a variety of southern minority groups.

In the third section, Morrison includes Beale's seminal 1975 article on the rural population revival of the 1970's that challenged many previously held assumptions about the country's migration and growth patterns. Because Beale personally knew the explanation for the growth in a particular area, he was able to show that no one cause explained this shift and that growth had not occurred evenly in all regions or even within regions. Another chapter elegantly sketches the sweeping changes in the farm population, with transformations in age patterns, family size, succession and migration, and why these changes have come about. Beale's more recent concern with the social geography of rural poverty is illustrated in two chapters that examine the relationship among natural resources, the rural economy, and particularly, persistent minority poverty. Showing the spatial relations of rural employment and natural resources in a series of county maps makes a striking impact not possible in a table. It illustrates Beale's ability to translate numbers into a short and pungent message.

The record of Beale's career provides evidence to those who rely only on macrodata for the value of onsite investigation. What is behind numbers is the reality of people who make the choices that generate trends. Beale's encapsulated record of rural America is well worth a read by anyone who is fascinated by the diversity and shared features of its past and present.

## Not for the Cynic: A Gentle Approach to Personal Discovery

*Three Faces Of Power.* By Kenneth E. Boulding. Newbury Park, CA: Sage Publications, 1989, ISBN 0-8039-3554-4, 259 pages, \$28.

Reviewed by Dwight M. Gadsby

Professor Boulding has arrived at a rare and exalted position where, as an author, he becomes almost as interesting as the subject matter he explores. He has been described as one of the "magistral" figures of the social sciences, and he is. Boulding himself represents a number of important confluences unique in economic science today. Although denying that he is a strict Keynesian, he may have more in common with that body of thought than any other. He appears to have important ties with the earlier "New Economics" practitioners, such as Professors John R. Hicks and Nicholas Kaldor. It is important to remember that at the age of 31, he made economic theory readable in *Economic Analysis*. Several later editions remain as touchstones for economists who take economic theory seriously.

*Three Faces of Power* is an essay about power, all kinds, and its use and misuse. The reader will find the book devoid of mathematical symbols and complex formulas, which are commonly used today to condense relationships and concepts. Some might even go so far as to question where the economic relationships are located. The book deals with ultimate economic questions and issues, such as the fragile nature and the organization of the international economy. Boulding confronts the value of short-term domestic and international gains relative to tradeoffs in terms of the life and death of the planet.

Boulding's life appears to be an affirmation of turning economics from a dismal science into a moral one. He approaches the writing of this book as an economist, a philosopher, and a poet. His words dance with a delicate subtlety that convinces the reader to read it several times. In his impeccable style, little moral lessons drift in softly: "Benevolence seems easier to express than does malevolence. Smiles take fewer muscles than frowns. It is harder to injure someone than to assist them." And those who expect a narrow interpretation of the use of power in the same sense Galbraith might employ will be disappointed. Boulding is much more subtle and philosophic.

Boulding centers his essay on power and divides it into three major categories: threat power, which can be fashioned into destructive features, particularly when

applied to political life; economic power, which rests on the capacity to function in the marketplace, especially in the ability to produce goods and services, based on the distribution and ownership of the factors of production; and integrative power, the power that emanates from personal relationships based on love and identity.

Boulding creates a setting from which to analyze threat power, which is effective only if it can be backed with economic and integrative power. Some of the thinking presented here may relate to his earlier analysis of macroeconomic aggregates. Boulding used some of these constructs when he supported the anti-nuclear movements in the 1950's and 1960's. I remember that in some of his rationale for disarmament in the 1950's, he invoked the threat of global destruction and the alarming exploitation of natural resources for short-run international political gains. His concern for the protection of natural resources and the environment marked him as a pioneer environmentalist. Today, he appears more uncertain about not confronting evil with force no matter the nature of threat. I wonder whether Boulding has changed since the 1950's when he opposed nuclear threat response, which was well on its way to providing the world with one of its longest periods of peace in this century. In this essay, he contends that any such deterrent can only be of short-term duration because if the probability of use is zero, then, in effect, it is no deterrent. But, the probability of global nuclear response has not been zero. Even if this probability is near zero, the tactic can be and has been of more than short-term duration due to the high impact of response and related consequences. A nuclear response has been arguably of low probability, but this seemingly low-probability event still carries dire consequences.

But have Boulding's views changed in the face of the need to confront unprovoked aggression with military and economic power, even as acquiescence threatens universally held values? Probably not. His essay is less about technological potentials and more about the hopes of people to adapt and cooperate. What would Boulding say about the events in the Persian Gulf, the international cooperation against aggression, the causes and aftermath? Even if Kuwait had been applying Boulding's power principles (economic and integrative power in anticipation of an invasion by an outside force), Iraq, with comparable economic and integrative power, would still have invaded.

Boulding holds out great hopes for the human spirit. Anatol Rapoport has said that Boulding sees science not as a system of theories, but as an ongoing struggle of human enterprise, a triumph of the human spirit,

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and a passionate search for wisdom. Boulding tells us not to be frightened to be human and not to fear failure. I wish he would have written more about the economics of compassion and how investment rates and opportunities could be integrated with the economic systems of this world.

Could Boulding help resource economists deal with questions about aggregate investments in human, social, and natural resources, both domestically and internationally? He reminds us that there is a profound ignorance about how a total world system would operate. He points out the need to overcome the stability of "poverty subcultures" as well as to explore the possibility of another "Great Depression." He asks us to consider the limits of world governments and their ability to deal with world problems. He wonders if there may be "positive-feedback" systems in operation that might destabilize a world economy and lead to a global disaster. Boulding is really stating that we live in a world that is a total system with nobody in charge. He sees that we must eventually deal with the reality of a "carrying capacity" for the earth's riches. He implies that a need exists to reorganize our disci-

plines so that social scientists can address many unanswered questions.

The value of reading Boulding's essay is that of personal discovery. Besides the many glittering economic ideas and suggestions lying about like diamonds in a field, his essay provokes self-knowledge. Boulding is unashamed of advocating "hugs" over "clubs" in solving international economic problems. In reading Boulding, I wondered what he learned from James A. Hobson's *Imperialism: A Study*, and did Boulding really know what the British Empire was all about? I would like him to have addressed the issues of international demographic models, using them to describe international macroeconomic aggregates. Also missing are his estimates of the finite limits and safety zones for the international economy. Boulding might have included some of those things he does so well, the nuts and bolts of economics, such as economic structure, income relationships, price flexibility, and international relationships between price and wage levels. But, despite these lapses, you as an economist will know a lot more about yourself and the failures and successes of your profession after reading this book.

## American Journal of Agricultural Economics

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**Articles:** Garth J. Holloway, "The Farm-Retail Price Spread in an Imperfectly Competitive Food Industry"; John Schroeter and Azzeddine Azzam, "Marketing Margins, Market Power and Price Uncertainty"; Fred J. Ruppel, Fred O. Boadu and E. Wesley F. Peterson, "Federalism, Opportunism and Multilateral Trade Negotiations in Agriculture", Kenneth H. Mathews and Duncan M. Holthausen, Jr., "A Simple Multiperiod Minimum Risk Hedge Model"; Daphne S. Taylor and Truman P. Phillips, "Food Pricing Policy in Developing Countries: Further Evidence on Cereal Producer Prices", Bruce A. Larson and Mary K. Knudson, "Public Regulation of Agricultural Biotechnology Field Tests: Economic Implications of Alternative Approaches"; Yir-Hueih Luh and Spiro E. Stefanou, "Productivity Growth in U.S. Agriculture Under Dynamic Adjustment", Julian M. Alston and James A Chalfant, "Unstable Models from Incorrect Forms", Julie A. Nelson, "Quality Variation and Quantity Aggregation in Consumer Demand for Food"; plus other articles, comments, and book reviews.



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# Imperfect Competition and Political Economy in Trade

***Imperfect Competition and Political Economy: The New Trade Theory in Agricultural Trade Research.*** Edited by Colin A. Carter, Alex F. McCalla, and Jerry A. Sharples. Boulder, CO: Westview Press, 1990, 270 pages, \$32.95.

***Reviewed by Daniel Pick***

Read this book if you are thinking about using imperfect competitive or political economy models in your trade and policy analysis. It contains thorough references on the most recent developments in these trade theories.

The 1989 summer symposium sponsored by the International Agricultural Trade Research Consortium was devoted to new developments in trade theory and implications for agricultural trade research. Seven papers from the symposium became chapters of the book, which is divided into two main themes. The first, in pieces by Krishna and Thursby, Richardson, Thursby and Thursby, and MacLaren, pinpoints issues of imperfect competition and international trade. The other, in pieces by Moore, Ray, and de Gorter and Tsur, examines political economy and its implications and trade modeling.

The opening two chapters extensively review literature on the current theory of international trade and imperfect competition (Krishna and Thursby), as well as survey the empirical estimates and approaches in the literature (Richardson). The surveys are weak in their limited applicability to international agricultural trade research. Annania, discussing Krishna and Thursby's paper, says of the existing theoretical models of imperfect competition: "Both settings appear far from allowing the extension of the results reached to international agricultural market policy analysis."

Richardson's writing on the existing empirical estimates of measuring national welfare contains a long reference list that emphasizes the literature on general equilibrium models. Empirical applications of computable general equilibrium models have gained popularity among agricultural economists who find them useful for policy analysis.

Thursby and Thursby furnish an empirical application of a duopoly model to Canadian and U.S. wheat exports to Japan by using an industrial organization approach in modeling wheat trade. This type of reconciliation between international trade and industrial organization has been applied recently in many empirical studies. Industrial organization models offer an alternative approach to investigate the role of market structure in international trade. Helpman and Krugman<sup>1</sup> noted that decisionmakers in agriculture may take advantage of the pervasive role of institutional arrangements to break away from price-taking behavior. They advocate more fully exploiting the link between international trade and models of industrial organization. The shortcomings of this type of approach, as used by Thursby and Thursby, are covered in the discussion by Veeman.

MacLaren discusses the issue of differentiated products as applied to international trade theory in general and agricultural trade in particular. He mentions the weakness of using the Armington model, pointing to other approaches that can be used in modeling international trade in differentiated products.

The book's other segment on political economy of trade contains two survey papers by Moore and Ray and ends with a structural model of political economy and trade for agriculture presented by de Gorter and Tsur.

The book contains: (1) "Introduction" by Colin A. Carter and Alex F. McCalla; (2) "Trade Policy with Imperfect Competition: A Selective Survey" by Kala Krishna and Marie C. Thursby; (3) "International Trade, National Welfare, and the Workability of Competition: A Survey of Empirical Estimates" by J. David Richardson; (4) "Strategic Trade Theory and Agricultural Markets: An Application to Canadian and U.S. Wheat Exports to Japan" by Marie C. Thursby and Jerry G.

Thursby; (5) "Implications of New Trade Theory for Modelling Imperfect Substitutes in Agricultural Trade" by Donald MacLaren; (6) "New Developments in the Political Economy of Protectionism" by Michael O. Moore; (7) "Empirical Research on the Political Economy of Trade" by Edward John Ray; (8) "The Political Economy of Agricultural Policy and Trade" by Harry de Gorter and Yacov Tsur; (9) "Summary" by Stephen L. Haley and Jerry Sharples.

Pick is an agricultural economist with the Agriculture and Trade Analysis Division, ERS.

<sup>1</sup>E. Helpman and P.R. Krugman, *Trade Policy and Market Structure*, The MIT Press, Cambridge, MA, 1989.



Moore summarizes the new developments in the theory of political economy, while Ray surveys the empirical research literature on the political economy of trade. Since both surveys are conducted by economists not necessarily familiar with agricultural policies, it is left to the reader to try to link the existing models to the reality of agriculture.

At its conclusion, the book develops a structural model of the political economy of agricultural policy and trade. De Gorter and Tsur offer a model in which pol-

iticians and parties compete for votes. The model then tries to explain the patterns of trade and policies. The model, however, suffers from some shortcomings that are well addressed in the discussion by Honma.

Overall, I recommend the book to those who are interested in research applied to international trade and the determinants of policies. In particular, the reader will find the survey chapters most useful in exploring the new theories of imperfect competition and political economy.

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# U.S. Grain Policies Viewed From Down Under

*U.S. Grain Policies and the World Market.* By Ivan Roberts, Graham Love, Heather Field, and Nico Klijn. Policy Monograph No. 4. Australian Bureau of Agricultural and Resource Economics, Canberra, 1990, xvii + 286 pages, \$39.95.

*Reviewed by Robert C. Green*

The United States, according to the authors, talks of market orientation and trade liberalization while providing government support for grain production and exports. The authors agree that by lowering price support levels, the United States has made strides toward a more market-oriented policy. At the same time, there are those who lobby for continued support of agricultural production and exports and against policies promoting market orientation and trade liberalization. The authors encourage the United States to continue moving its agricultural policies toward market orientation.

The authors invite a wide audience by presenting their arguments in nontechnical language and locating the supporting analysis and more technical concepts in appendixes. They describe objectives and provisions of U.S. grain policy since 1950, characterizing by period according to policy objectives, approaches, and market conditions. Then they focus on the international effects of U.S. grain policies and other forces affecting world trade. The authors discuss the economic and political factors that affect U.S. grain policies and assess the performance of those policies in meeting their objectives of income support, price stability, and export market share. The paper concludes with policy alternatives open to the United States.

The authors peak with their presentation of international implications and forces affecting U.S. grain policies, providing a useful discussion of the history of grain markets and market shares and evaluating factors that affect wheat exports. They note the importance placed on export market shares in the making of the Food Security Act of 1985 but question whether any benefits were realized in the process.

They show the level of U.S. stockholding relative to world stocks and discuss stockholding policy and its effects on price stability. Stocks accumulated in the early 1980's, termed excessive, led to provisions in the Food Security Act of 1985, including the Export Enhancement Program (EEP). They discuss the effects of EEP on world trade and competitors' response to the program.

Three points focus on the performance of U.S. policies: (1) Although price stability has been a goal of U.S. policies, it has not been sustained over the long term. (2) Price reduction and destocking policies followed in the 1980's have regained export market shares but have not recovered export revenues. (3) U.S. grain policy is so complex and interactive that it is impossible to evaluate the costs and benefits of various provisions. I agree with the authors on the first two points. I also agree that grain policy is complex and interactive. But given a set of assumptions, we can evaluate the costs and benefits of various provisions.

The authors furnish an evaluation of policy alternatives for the United States, which, they believe, extends market power only to the United States. The alternative policies presented are classified as those which use present mechanisms, those which optimize benefits to the U.S. economy or to grain producers, or those which are practical reform options. This analysis demonstrates how alternative policies may benefit or cost U.S. producers, consumers, taxpayers, and the national economy, and foreign producers and consumers. The policy that provides the most benefit to the U.S. producer may not be the policy that provides the most benefit to the U.S. economy. The authors use a clearly detailed, informative static analysis to present structural effects of policy.

The authors provide a description of U.S. institutions and policy development, presenting the players in the process and how they operate. This is an important section which explains to general readers how politics cause or fail to cause change. The analysis falls short as an overall presentation of U.S. policy. The authors fail to form a comprehensive look at U.S. grain policies. They never really explain how the various components of grain price and income support programs interact.

The focus on nonrecourse loan activity and its price effects oversimplifies how U.S. policy operates. The authors introduce farmer-owned reserves, certificates, and the EEP and their price effects, but never fully describe the programs or cohere them. A general audience may not understand the objectives, structure, and operation of grain policy, and thus would not fully appreciate the points and issues of the authors.

In summary, the authors provide a good presentation of issues concerning U.S. grain policy and its effects on world trade. They argue for continuing market-oriented policy and trade liberalization. Their discussion of the politics of policy and why change is difficult is also useful. However, they do little to clarify U.S. grain policy and its operation.

Green is an agricultural economist with the Agriculture and Trade Analysis Division, ERS.



## Plotting Agricultural Policy Means Changing Course

*The Political Economy of U.S. Agriculture: Challenges for the 1990s. Edited by Carol S. Kramer. Washington, DC: Resources for the Future, 1990, 298 pages, \$20.*

*Reviewed by David Ervin*

Agricultural and food policy is an enigma to many. Through sometimes conflicting government programs, the public sector often influences the agricultural and food industries to achieve diverse goals. Not surprisingly, even policy analysts have difficulty identifying the effects of policy on food production, prices, trade, the environment, and other processes, let alone predicting the path of future policy. The papers in this book help explain how the web of current agricultural policies was woven, and how to identify the major forces that will influence new policies. It offers provocative and productive reading for agriculture and food policy analysts.

The authors present the most complete coverage of the current farm and food policy mix. Articles are well written and argued with considerable insight, but unfortunately, with little empirical analysis to support strong conceptual arguments in most cases.

Much of the work, while academically interesting, does not push beyond how we got to where we are, or venture analyses of policy changes that might accompany a changing agricultural and food agenda. To be fair, the editor explains that the book is not intended to explore new policy directions, but rather to "examine the political economy in which agricultural and food policy is formulated today in the United States." Therefore, this volume is most useful in providing an understanding of the evolution to the current policy setting, a commendable exercise that preceded construction of the recently passed 1990 farm bill and the still emerging GATT agreement.

Four important strains of thought inform the analysis: increasing roles of external factors, international dimensions of policy, policy challenges of new technology, and contributions of policy analysis.

Agriculture and food policies are increasingly influenced by forces outside the traditional agricultural policy domain, including the environment, food safety, biotechnology, international trade, and the budget deficit. Partial evidence from the 1990 farm bill process supports the "external forces" premise. The most

important reform in the 1990 Food, Agriculture, Conservation, and Trade Act is the increased flexibility in commodity program plantings, a direct result of budget deficit pressure and perhaps of GATT strategies. Agricultural policy analysts should sort these outside influences by degree of importance. Such a ranking will not only help identify the likely path of policy evolution but will better target the policy research agenda. For example, the effect of budget and trade pressures appears to supersede environmental, food safety, and technology concerns. In effect, the more important external forces determine the feasible set of policy choices for environmental, food safety, and other issues.

Global forces are profoundly affecting agricultural and food policies. Examples cited in the analyses range from immigration policies for farmworkers to international technology transfer to trade agreement negotiations. When the history of agricultural development in the 1990's is written, the increasing internationalization of U.S. agriculture will be stressed. This "opening" of the agricultural industry will not likely be reversed. The integration of monetary systems with large international capital investment is a powerful force. Still other international influences may emerge. Pressure mounts to have international agreements on significant global resource changes. This book illuminates this important international dimension.

The book includes: (1) "Introduction and Overview" by Carol S. Kramer; (2) "Why Is Agricultural Policy So Difficult To Reform?" by James T. Bonnen and William P. Brown; (3) "Is There Anything 'American' About American Agricultural Policy?" by Robert L. Paarlberg; (4) "Selective Perceptions and the Politics of Agricultural Policy" by James Duncan Shaffer; (5) "Agriculture and the Failure of the Budget Process" by Charles H. Riemenschneider and Robert E. Young II; (6) "The Political Economy Of Farm Credit Reform: The Agricultural Credit Act of 1987" by David Freshwater; (7) "The Evolution of Pesticide Policy: Environmental Interests And Agriculture" by Katherine Reichelderfer and Maureen Kuwano Hinkle; (8) "Biotechnology and Agriculture in the Congressional Policy Arena" by L. Christopher Plein and David J. Webber; (9) "Food Safety and International Trade: The U.S.-EC Meat and Hormone Controversies" by Carol S. Kramer; (10) "Farm Workers, Agriculture, and the Politics of Immigration Reform: The Immigration Reform and Control Act of 1986" by Rekha Mehra; (11) "Choices and Challenges for the 1990's" by Carol S. Kramer, Barbara J. Elliott, Lawrence M. Rubey, and George E. Rossmiller.

Ervin was a branch chief in the Resources and Technology Division, ERS. He recently accepted a position in the Department of Agricultural Economics, Oregon State University, Corvallis.

Several authors comment on the forces for change created by new technologies. Their comments reflect the powerful technological process that agriculture has undergone and the potentially greater technological revolution ahead. But, technology today is a double-edged sword. Many emerging technologies offer the promise of productivity gains but often raise the specter of health and environmental problems. The writings recommend understanding the public's fear of new technologies like the bovine growth hormone. Allaying those fears has proven difficult despite strong countervailing scientific evidence. Such a recommendation may help salvage potentially important productivity improvements that may offer lower costs and safer food and fiber products. In a closing comment, the editor stresses the need for empirical analysis to document possible external effects of technologies and to characterize tradeoffs of policy approaches. The book analyzes technology as exogenous to agricultural and food policy. Possibilities for understanding the economic and political forces that endogenously determine the path of technological change are left largely unexplored. Both private and public technological

developments are biased by the absence of prices for nonmarket environmental services and by public commodity programs. Understanding and documenting the nature and magnitude of those biases may help in understanding public fears about new technologies.

The editor issues five challenges to policy analysts in the agricultural and food policy process: (1) examine why things (policies) are the way they are, (2) use inputs from multiple disciplines, (3) address the role of policy institutions and processes in determining outcomes, (4) educate through policy analysis, and (5) explore conscious or unconscious assumptions in policy analyses. Perhaps an implicit element in some of the challenges needs amplification. The most useful policy contributions come from the best, policy-relevant scientific analyses. Studies of historical or current policy often focus on the evolution of policy instruments. But those instruments are products (or symptoms) of underlying problems. Understanding the driving forces in those problems through sound conceptual and empirical research yields the best policy analysis.



## Risk Analysis/Risk Management

***Risk Analysis: A Guide to Principles and Methods for Analyzing Health and Environmental Risks.*** By John J. Cohrssen and Vincent T. Covello. U.S. Department of Commerce, The National Technical Information Service, 1989, 407 pages, \$17.50.

***Reviewed by Michael E. Wetzstein***

Interested in a handbook that explains how to develop an analysis for health and environmental risks? Then this guide by Cohrssen and Covello is it. Published by the Council on Environmental Quality, this guide is an accessible reference manual for educators and researchers in the area of risk analysis. Cohrssen and Covello write in clear language, understandable to both specialists and nonspecialists.

The introductory chapter discusses the justification for risk analysis, clearly distinguishing among the terms risk assessment, analysis, and management. With this foundation, Cohrssen and Covello outline a process of analyzing health and environmental risk. They identify four interrelated phases (hazard identification, risk assessment, determining the significance of risks, and risk communication) for risk analysis. After presenting an overview of risk analysis in chapter two, they devote the remaining chapters to discussing each phase.

Chapter two is somewhat redundant, as basic definitions of terms are again defined but in greater detail. However, useful graphics highlight the dimensions of risk, the effects on risk perception, and risks addressed by each Federal statute. Society's perceptions and Environmental Protection Agency priorities, benefit-cost analysis, and *de minimis* risk are three issues discussed at the end of this chapter.

Cohrssen and Covello compare epidemiological studies, *in vivo* animal bioassays, short-term *in vitro* cell and tissue culture tests, and structure-activity relationship analyses as techniques for hazard identification. Case studies of Woburn, Massachusetts, water contamination and dioxin chemical compounds illustrate hazard identification. The proper interpretation of community health studies and the role they play in hazard identification are also addressed. Throughout the guide, detailed definitions of terms are provided. For example, a section on carcinogenicity characterization compares the criteria used for identification of carcinogenicity by the U.S. Environmental Protection Agency, International Agency for Research on Cancer, National Toxicology Program, and the American Conference of Governmental Industrial Hygienists.

A major controversy in hazard identification is the use of large doses in carcinogenicity tests. Cohrssen and Covello respond by detailing the advantages and disadvantages of various techniques: source/release, exposure, and dose-response. Cohrssen and Covello describe monitoring and modeling techniques used in these procedures. Risk characterization designed to generate estimates from the results of source/release, exposure, and dose-response are also discussed. Limited discussion is provided on ecological risk assessment and identifying and evaluating uncertainties in risk estimates. However, as in all sections, Cohrssen and Covello provide supplementary information for a student with further interest in this area.

In the last chapter, Cohrssen and Covello discuss the problems of risk communication, which they define as any purposeful exchange of information. In accordance with other phases, a multitude of problems beset the risk analyst. Unfortunately, this leaves one with a lot of unanswered questions. Analysis of health and environmental risks is relatively new, and theories and techniques are still under development. In many sections, Cohrssen and Covello establish the foundation for further research by outlining problems that still complicate any risk analysis. Also, the Environmental Protection Agency's cardinal rules of risk communication are listed as a guide for effective communication.

Approximately three-fourths of the book is in appendixes. Appendix A summarizes test systems and assays commonly used to evaluate whether a chemical, radioactive, or biological agent poses a hazard to human health or the environment. As an example, the *Salmonella* Mutagenicity Assay (Ames Test) is discussed in terms of endpoints measured, effects inferred, protocol summary, major sources of uncertainty, accuracy, degree of development, and resources required. Publications of the International Agency for Research on Cancer, and chemical and physical agents for which there is evidence of carcinogenicity to man, are listed in appendixes B and C. Each agent is weighed evidentially, in terms of: sufficient evidence to establish a causal relationship, limited evidence for a causal relation, inadequate evidence, and no evidence available for humans and animals.

Appendix D describes various mathematical expressions and units of measurement used in risk analysis, and standards and recommended criteria for pollutants are listed in appendix E. Regulations for implementing the procedural provisions of the National Environmental Policy Act are reprinted in appendix F. A 1985 review by the Office of Science and Technology Policy on the science associated with chemical carcinogens is reprinted in appendix G. This review contains an

Wetzstein is a professor in the Department of Agricultural and Applied Economics, University of Georgia, Athens.

extensive list of references for further development of risk analysis guidelines.

The last appendix reprints the Environmental Protection Agency's risk assessment guidelines related to carcinogen mutagenicity, chemical mixtures, suspect developmental toxicants, and estimating exposures. As a source for quick reference, following the appendixes is a list of acronyms, abbreviations, and a glossary of risk analysis terms.

This guide provides an excellent foundation for any student interested in learning how risk analysis is currently undertaken. It is written in a clear nontechnical language requiring little or no scientific background. In a classroom setting, the guide would complement lectures in basic theory by providing students with an understanding of the practical problem of implementing this theory. Courses in the social sciences (including economics and psychology), physical sciences, and biological sciences related to health and environmental risks will significantly benefit from this guide. However, in a multidisciplinary curriculum directed toward environmental studies, this guide would serve as an integral part of a capstone course.

In public service, where educating the citizen is an integral part of governing, Cohrssen and Covello have provided clear and concise reasons for many current techniques employed for risk analysis. They establish a foundation for communicating with concerned citizens the current and future policies and programs of governing agencies.

***Agricultural Risk Management.*** By Beverly Fleisher. Boulder, CO: Lynn Reinner Publishers, Inc., 1990, 149 pages, \$25 (hardcover).

Risk colors all decisions. This is particularly true in the agricultural sector, which is susceptible not only to changes in market conditions, but also to environmental and governmental policy uncertainty. Agricultural decisionmakers employ a broad range of techniques to manage this agricultural risk. An understanding of how these risk management techniques are interrelated can aid in the development of an effective agricultural policy addressing the risky nature of agriculture.

In contrast to other books on risk management that discuss management techniques within a narrow range of decisions, Fleisher attempts to integrate alternative risk management techniques with the whole set of agricultural decisions. Her objective is not to provide an exhaustive and rigorous treatment of each risk management technique. Instead she maps out a general understanding of alternative risk management techniques accompanied by a discussion of their inter-

relationships. A list of relevant literature closes each chapter.

Fleisher's book is intended for nonspecialists in risk and risk management. Fleisher guides the reader through agricultural risk, decisionmakers' responses, risk management techniques, and how government programs affect risk and risk management. She stresses the interaction of decisionmakers and government policies with risk management, and she discusses the possibility of substituting privately sponsored risk management programs for current government programs. The book is an excellent supplement for classes in agricultural policy, finance, and marketing, requiring just basic knowledge of economics and finance as a background to understanding the material presented. This is not a definitive work. For example, Fleisher fails to address environmental considerations associated with risk management; and food safety and water contamination issues are not investigated. Current public concern with these issues warrants a discussion of risk management options in these areas.

Some technical terms (for example, "noise in price signals") creep into the text that require an explanation for a nonspecialist. Many of these phrases have alternative definitions, and thus can be confusing to the reader. Fleisher does offer an excellent discussion of alternative definitions of risk and uncertainty. However, this discussion is relegated to chapter two, leaving a student to wonder what is meant by risk and uncertainty throughout the first chapter. A number of examples illustrate the various concepts of risk, like calculating the expected value and outcome of a lottery. Examples of some less obvious results would be useful too. For instance, illustrating how a risky decision that complements other producers' activities will actually reduce overall risk would provide a student greater understanding of risk management.

Fleisher makes an analogy to murder mysteries, citing the temptation to skip to the end to solve the mystery. Unfortunately, in this book, Fleisher provides no such solution, and the reader is left with the unsolved mystery of determining the "best" risk management policy. This is not so much the book's shortcoming as a failure of economic theory to provide a definitive set of policies to improve society. Each theory provides only the tools and procedures sufficient to solve the mystery. It is up to the student, in each case (mystery), to employ the relevant theory and determine the "best" policy (solve the mystery).

Overall, this is an excellent book on agricultural risk management. Fleisher supplies a very sound and readable discussion of how alternative risk management decisions are integrated. I recommend this book not only to nonspecialists but to specialists as a clear and concise discussion of agricultural risk management.



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The Agricultural Economics Society

Conference Programme, Nottingham, April 1990

Annual General Meeting: Minutes of the 60th

Prize Essay Competition

**EDITOR: Professor K. J. THOMSON**

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